# Bayesian Stochastic Mortality Modelling for Two Populations


## Abstract:

The paper introduces a new framework for modelling the joint development over time of mortality rates in a pair of related populations by combining a number of recent and novel developments in stochastic mortality modelling. First, we develop an underlying stochastic model which incorporates a mean-reverting stochastic spread that allows for different trends in mortality improvement rates in the short-run, but parallel improvements in the long run in line with the principles of biological reasonableness. Second, we fit the model using a Bayesian framework that allows us to combine estimation of the unobservable state variables and the parameters of the stochastic processes driving them into a single procedure. This procedure employs Markov chain Monte Carlo (MCMC) techniques, permitting us to analyse uncertainty in the estimates of the historical age, period and cohort effects, and this helps us to smooth out noise in parameter estimates attributable to small populations.

Mortality rates arising from this framework provide consistent forecasts for the two populations, reflecting not just parallel sample paths in the long run, but also similar levels of variability around the central trend. Further, estimated correlations based on the simulated mortality improvement factors for two populations are consistent with historical data.

The framework is illustrated using two-population extensions of the Age-Period-Cohort and Lee-Carter models on the following populations: England & Wales national and CMI assured lives males and females, and US and California males. The approach is designed for large populations coupled with a small sub-population, but is easily adaptable to other combinations.

A key application of the modelling framework would be to allow longevity risk hedgers to analyse the basis risk that exists between mortality rates in two populations in the case where the hedger wishes to hedge the risk in one population using an index hedge based on the second population.



**Keywords**: Stochastic mortality, two populations, small sub-populations, mortality spreads, age effect, period effect, cohort effect, basis risk, Markov chain Monte Carlo, parameter uncertainty, limited data, missing data, fan charts.



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