The Poisson log-bilinear Lee Carter model: efficient bootstrap in life annuity actuarial analysis

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In life insurance, the companies deal with two fundamental types of risks when issuing contracts: financial risk and demographic risk. As regards the latter, the uncertainty of the future developments of the mortality is of particular importance to financial institutions involved with selling annuities and pensions-related products. Recent work has focused on modelling the trend in mortality as a stochastic process.

The most popular method for modelling the death rates is the Lee Carter model (1992), because it is easy to implement and outperforms other models with respect to its prediction errors (e.g., Koissi et al., 2006; Melnikov and Romaniuk, 2006).

This methodology has become widely used and there have been various extensions and modifications proposed to attain a broader interpretation and to capture the main features of the dynamics of the mortality intensity, e.g. the version log-bilinear Poisson Lee Carter model as in Renshaw and Haberman 2003. In particular, in order to improve the survival probability outputs from the point of view of the longevity impact, the most recent approach is based on simulation procedures applied to the Lee Carter family models (Brouhns et al., 2005 e D'Amato et al., 2009). These studies provide confidence intervals for forecasted quantities derived by using simulation techniques: these are important because of the non-linear nature of the quantities under consideration. Both are based on the bootstrap methodology for obtaining more reliable and accurate mortality projections and are semi parametric bootstrap from the Poisson distribution. In particular, the latter is based on the idea of obtaining an acceptable accuracy of the estimate by solving the problem of reducing the variance estimator by means of the VRT's (Variance Reducing Techniques).

This demographic framework allows for identifying three different demographic basis in which the survival behaviour is accurately described taking into account the longevity contribute as function of the time of observation and in its evolution in the time horizon.

In this order of ideas, the longevity question constitutes an incisive element in the pension annuity solvency appraisal, as a wide literature explains (see for example Hari et al., 2008 and Olivieri and Pitacco, 2003). The demographic system modelling a portfolio cash flow distribution impacts on the mathematical reserve and surplus and orientates the choices of the pension plan management (Coppola et al., 2007, D'Amato et al., 2009). In the life annuity schemes the valuations are performed at the issue time on the basis of the financial and demographic information flow at disposal at that time and are intensely sensitive to the longevity phenomenon strength. It derives the crucial matter of the survival modelling in order to obtain, from the pension plan point of view, the estimation of premiums to receive during the accumulation period, from the beginning to the retirement age, and of benefits to pay during the annuitization period, from the retirement age until the contractor is alive. Both premiums and benefits feels the demographic description in the two aspects of their quantification and of the number of them to pay or to receive.

The paper deeps the impact of the survival modelling uncertainty in life annuity portfolios in a context in which the interest rate structure is stochastic. In particular it provides mathematical reserve and surplus values for pension annuity portfolios in order to perceive the real contribute of the considered refined methodologies in actuarial entities that can immediately be interpreted and used to the solvency aim.

Keywords: life annuity, longevity risk, stochastic mortality, stochastic interest rates, mathematical reserve, actuarial surplus, bootstrap, Variance Reducing Techniques.

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