Risk and Valuation of Mortality Contingent Catastrophe Bonds
Daniel Bauer & Florian Kramer
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  → pricing models for mortality-contingent securities in incomplete market framework
  → model for mortality index: GBM & multiplicative jump component

- Chen & Cox (2009, JRI)
  → pricing models for mortality-contingent securities in incomplete market framework
  → Lee-Carter extensions with multiplicative jump component

- Cox, Lin & Milidonis (2009)
  → pricing models for mortality-contingent securities in incomplete market framework
  → regime-shifting models
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  - pricing models for mortality-contingent securities in incomplete market framework
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  - pricing models for mortality-contingent securities in incomplete market framework
  - regime-shifting models

- Here:
  - risk assessment, comparison to official loss profiles
  - "endogenous" valuation (similar to Lin & Cox (2005,JRI); Bauer, Börger & Ruß (2009,IME)) → problems...
  - affine mortality model with additive jump components
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### Vita Capital Ltd.
- **Issued**: Nov. 2003
- **Class**: A, B
- **Tranche Size**: $400mn, $62mn
- **Arranger**: Swiss Re
- **Protection for**: Swiss Re
- **Rating**: A3/A+, Aa3/A-**
- **Attachment Point**: 130%
- **Detachment Point**: 150%
- **Coupon (bps)**: LIBOR+135
- **Expected Maturity**: 4 years
- **Covered Area**: US 70%, UK 15%, F 7.5%

### Vita Capital II Ltd.
- **Issued**: Apr. 2006
- **Class**: C, D
- **Tranche Size**: $200mn, $100mn
- **Arranger**: Swiss Re
- **Protection for**: Swiss Re
- **Rating**: A2/BBB+, Baa2/BBB-**
- **Attachment Point**: 120%
- **Detachment Point**: 125%
- **Coupon (bps)**: LIBOR+90
- **Expected Maturity**: 5 years
- **Covered Area**: US 62.5%, UK 17.5%, D 7.5%, CAN 5%

### Tartan Capital Ltd.
- **Issued**: May 2006
- **Class**: A, B
- **Tranche Size**: $75mn, $80mn
- **Arranger**: Goldman Sachs
- **Protection for**: Scottish Re
- **Rating**: Aaa/AAA, Ba3/BBB+
- **Attachment Point**: 115%
- **Detachment Point**: 120%
- **Coupon (bps)**: LIBOR+190
- **Expected Maturity**: 3 years
- **Covered Area**: US 100%

### Osiris Capital Plc.
- **Issued**: Nov. 2006
- **Class**: B1, B2
- **Tranche Size**: Euro 100mn, Euro 50mn
- **Arranger**: Swiss Re
- **Protection for**: AXA
- **Rating**: Aaa/AAA, A3/A-
- **Attachment Point**: 114%
- **Detachment Point**: 119%
- **Coupon (bps)**: EURIBOR+20
- **Expected Maturity**: 4 years
- **Covered Area**: F 60%, J 25%, US 15%

### Vita Capital III Ltd.
- **Issued**: Dec. 2006
- **Class**: C, D
- **Tranche Size**: $150mn, $100mn
- **Arranger**: Swiss Re
- **Protection for**: Swiss Re
- **Rating**: Baa2/BBB, Ba1/BB+
- **Attachment Point**: 110%
- **Detachment Point**: 114%
- **Coupon (bps)**: LIBOR+250
- **Expected Maturity**: 4 years
- **Covered Area**: US 62.5%, UK 17.5%, D 7.5%, J 7.5%, CAN 5%

### Vita Capital III Ltd. (cont.)
- **Issued**: Jan. 2008
- **Class**: BV
- **Tranche Size**: Euro 30mn
- **Arranger**: Munich Re, JPMorgan
- **Protection for**: Munich Re
- **Rating**: A1/A
- **Attachment Point**: 120%
- **Detachment Point**: 125%
- **Coupon (bps)**: LIBOR+110
- **Expected Maturity**: 5 years
- **Covered Area**: US 62.5%, UK 17.5%, D 7.5%, J 7.5%, CAN 5%

### Nathan Ltd.
- **Issued**: Feb. 2008
- **Class**: A
- **Tranche Size**: $100mn
- **Arranger**: Munich Re, JPMorgan
- **Protection for**: Munich Re
- **Rating**: A1/A
- **Attachment Point**: 120%
- **Detachment Point**: 125%
- **Coupon (bps)**: LIBOR+110
- **Expected Maturity**: 5 years
- **Covered Area**: US 45%, CAN 25%, UK 25%, D 5%

### SCOR Mortality Swap
- **Issued**: Jan. 2008
- **Class**: na
- **Tranche Size**: $100mn + Euro 36mn
- **Arranger**: SCOR
- **Protection for**: SCOR
- **Rating**: A2/A-
- **Attachment Point**: 115%
- **Detachment Point**: 125%
- **Coupon (bps)**: LIBOR+135
- **Expected Maturity**: 4 years
- **Covered Area**: US, Europe

### Table 3: Comparison of all CATM deals from 2003 until 2008 (Source: New Issue Reports from S&P and Moody’s; Bloomberg data).

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5 The tranches marked with * are guaranteed by monoline insurers. Most of these tranches were downgraded in 2008 due to financial trouble of the guarantors.

6 Rating at Issuance from Moody’s / S&P – the ratings marked with ** were upgraded by S&P.
Structure of a CATM transaction

- Combined Mortality Index $i_t$ contingent on the relative, weighted mortality experience of a certain population as reported from official entities in the years $t$ and $t - 1$:

$$i_t = \frac{1}{2} \left( \hat{q}_t + \hat{q}_{t-1} \right)$$

where

$$\hat{q}_t = \sum_{all \ x} \omega_{x,m} \hat{q}_{m,x,t} + \omega_{x,f} \hat{q}_{f,x,t}$$

- Loss Tranche $(a, d)$

$$l_t^{(a,d)} = \min \left\{ \max \left\{ l_{t-1}, \frac{i_t - a}{d - a} \right\}, 100\% \right\}$$

with $l_{2006}^{(a,d)} = 0$, a Attachment Point (e.g. 110%), $d$ Detachment Point (e.g. 120%).
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Stochastic Mortality Modeling

- Array of stochastic mortality models available: Cairns et al. (2006, 2007,...)


  → need jumps

- Here: → want, parsimonious affine structure ...no suitable candidate
  → "coherent" specification

(Yet another) model: Mean-reverting or not mean-reverting? Trend?
Rely on demographic data and research:
Positivity, Vaupel line (Oeppen & Vaupel (2002,Science)),
mortality spikes with additive influence
Stochastic Mortality Modeling

- Array of stochastic mortality models available: Cairns et al. (2006, 2007,...)

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- (Yet another) model:
  - Mean-reverting or not mean-reverting? Trend?
  ⇒ Rely on demographic data and research:
    Positivity, Vaupel line (Oeppen & Vaupel (2002,Science)),
Our model

\[
\mu_t(x) = e^{b(x+t)} Y_t + \Gamma_t
\]

where

\[
dY_t = \alpha \left( (Y_0 - \beta^{(2)}) e^{-\beta^{(1)} t} + \beta^{(2)} - Y_t \right) dt + \sigma \sqrt{Y_t} dW_t,
\]

\[
d\Gamma_t = -\kappa \Gamma_t dt + dJ_t, \quad J_t \text{ CPP with Exp. distr. jumps}
\]

"positivity",

"spikes"
Our model

\[ \mu_t(x) = e^{b(x+t)} Y_t + \Gamma_t \]

where \[ dY_t = \alpha \left( (Y_0 - \beta^{(2)}) e^{-\beta^{(1)} t} + \beta^{(2)} - Y_t \right) \ dt + \sigma \sqrt{Y_t} \ dW_t, \]

"positivity",

Solution to * disregarding stochastic part (ODE)

\[ \rightarrow t_{IP} \approx 44 \]

(inversion point)

\[ \circ \quad e_\infty \approx 86 \text{ years} \]

\[ \rightarrow \text{in line with demographic research} \]
Estimation

- Simulated Maximum Likelihood Estimation
- **Particle Filter** for Likelihood Evaluation
  ("Monte Carlo version of Kalman filter")
- **Issues:**
  - $p_x$'s depend not only on $\mu_x$, but on $[\mu_x, \mu_{x+1})$
  - Consider 4-dimensional state vector

\[
\begin{pmatrix}
Y_t, & \Gamma_t, & \int_t^{t+1} e^{b(s-t)} Y_s \, ds, & \int_t^{t+1} \Gamma_s \, ds \\
Z_t^{(1)}, & Z_t^{(2)}, & Z_t^{(3)}, & Z_t^{(4)}
\end{pmatrix}
\]

- regularity of likelihood function, local maxima in optimization
  - Sorry, Dr. Brockett!
  - used many many starting vectors based on smaller samples, final optimization with bigger sample
- **Pros:**
  - coherent ML estimation, model comparisons possible
  - "disentangle" jumps from continuous part, obtain distribution of states
Expected values of the states

- Allocation of initial improvements to jump component
- Clear jump event in 1918 → Spanish Flue
- Non-pandemic events noticeable (WWII or Vietnam war). Events affecting elderly population covered by cont. part

- Jumps necessary?
  - $p$-value of LR test essentially zero, Bayes factor $\exp\{1688.2\}$
  - Strong statistical evidence
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## Loss Profiles (Tartan deal)

<table>
<thead>
<tr>
<th></th>
<th>PD(%)</th>
<th>EL(%)</th>
<th>Spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cl. B Tranche</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jump Model, data 1901-2005</td>
<td>18.08</td>
<td>16.60</td>
<td>693</td>
</tr>
<tr>
<td>Model without jumps, data 1901-2005</td>
<td>9.69</td>
<td>3.35</td>
<td>111</td>
</tr>
<tr>
<td>Reported</td>
<td>0.88</td>
<td>0.54</td>
<td>–</td>
</tr>
<tr>
<td><strong>Cl. A Tranche</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jump Model, data 1901-2005</td>
<td>15.24</td>
<td>14.04</td>
<td>582</td>
</tr>
<tr>
<td>Model without jumps, data 1901-2005</td>
<td>0.47</td>
<td>0.12</td>
<td>4</td>
</tr>
<tr>
<td>Reported</td>
<td>0.29</td>
<td>0.16</td>
<td>–</td>
</tr>
</tbody>
</table>

- Risk measures exceed official profiles
- "Actuarially fair" spread for jump model exceeds market spread (300bps for Tranche B)
Estimation results based on 1950-2005 data

- lower volatility
- limiting life expectancy about the same
- basically no jumps
## Loss Profiles (Tartan deal)

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<tr>
<td><strong>Cl. B Tranche(110%-115%)</strong></td>
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<tr>
<td>Jump Model, data 1901-2005</td>
<td>18.08</td>
<td>16.60</td>
<td>693</td>
</tr>
<tr>
<td>Model without jumps, data 1901-2005</td>
<td>9.69</td>
<td>3.35</td>
<td>111</td>
</tr>
<tr>
<td>Jump Model, data 1950-2005</td>
<td>0.89</td>
<td>0.80</td>
<td>32</td>
</tr>
<tr>
<td>Reported</td>
<td>0.88</td>
<td>0.54</td>
<td>–</td>
</tr>
<tr>
<td><strong>Cl. A Tranche(115%-120%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jump Model, data 1901-2005</td>
<td>15.24</td>
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<td>582</td>
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<td>0.47</td>
<td>0.12</td>
<td>4</td>
</tr>
<tr>
<td>Jump Model, data 1950-2005</td>
<td>0.71</td>
<td>0.64</td>
<td>26</td>
</tr>
<tr>
<td>Reported</td>
<td>0.29</td>
<td>0.16</td>
<td>–</td>
</tr>
</tbody>
</table>

- Risk characteristics strongly depend on estimation period
- Investors’ beliefs in line with permanent regime change
"Endogenous" valuation

  - Derive risk-adjusted parametrization based on primary insurance prices – yields at least an upper bound for mortality derivatives
  - Fast calibration due to affine structure, results based on 73 term-life quotes
"Endogenous" valuation

- **Idea:** (cf. Lin & Cox (2005,JRI), Bauer, Börger & Ruß (2009,IME))
  - Derive risk-adjusted parametrization based on primary insurance prices – yields at least an upper bound for mortality derivatives
  - Fast calibration due to affine structure, results based on 73 term-life quotes

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**Baseline Component**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.018</td>
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<tr>
<td>$\beta^{(1)}$</td>
<td>0.0288</td>
</tr>
<tr>
<td>$\beta^{(2)}$</td>
<td>0.00006</td>
</tr>
<tr>
<td>$\sigma^*$</td>
<td>0.00026</td>
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<tr>
<td>$b^*$</td>
<td>0.083</td>
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<tr>
<td>$Y_0^*$</td>
<td>0.00016</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>1.033</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.114</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>211</td>
</tr>
<tr>
<td>$\Gamma_0^*$</td>
<td>1.49E+07</td>
</tr>
</tbody>
</table>

**Catastrophe Component**

- **What happened?**
  - lower baseline mortality: differences in populations, selection effects
  - permanent, small, high-frequency jumps: selection effects – approximation to ultimate mortality

→ These effects overshadow potential mortality risk premium. Risk measures basically zero.
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Conclusion

- Explain structure of CATM securitization, provide market overview
- Present time-continuous stochastic mortality model for the analysis of CATM bonds. With "only" eight parameters, our model...
  ... displays basic features that are in line with demographic data and research
  ... shows jumps that are structurally consistent with catastrophic mortality events observed in the last century
  ... offers a high degree of analytical tractability due to affine structure

- Primary result:
  Calculated risk profiles significantly exceed official loss profiles for most calibrations, large uncertainties \(\Rightarrow\) loss profiles should be interpreted very carefully by investors and rating agencies

- Future research:
  - extend model to multiple populations
  - can uncertainty aversion (ambiguity aversion) explain observed spread levels?
Contact

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Thank you!
Extra 1: Short calibration trend component

![Graph showing trend and beta^2 over years from 1950 to 2000.](image)