Living With Ambiguity: Pricing Mortality-linked Securities With Smooth Ambiguity Preferences

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Outline

Uncertainty In Mortality

- Mortality is a stochastic process: it is improving, to some extent, in an unpredictable way.
- We have imprecise knowledge about the probability distribution of future mortality rates.
- It seems appropriate to define mortality/longevity risk in a more general term of *ambiguity* in the sense of Knight (1921).

Risk	probabilities known random events.
Ambiguity	unknown probability assignment.

Uncertainty in Mortality Models

- Two kinds of uncertainty
  - Model misspecification
  - Parameter estimation
- Parameter uncertainty is particularly unavoidable in any model-based approach (Li and Ng, 2011)
- We think that parameter uncertainty has not be fully explored.

For instance,Cairns et al. (2006b) acknowledge ambiguity using Bayesian analysis, but treat it in an *ambiguity-neutral* way.

Outline

Ambiguity Aversion and Asset Pricing

- Thought experiments such as the famous Ellsberg paradox (Ellsberg, 1961) provide evidence that individuals generally prefer the least ambiguous acts.
- People usually exhibit ambiguity aversion, which can be thought as an aversion to any mean-preserving spread in the space of probabilities (Alary et al. 2010).
- If market participants are ambiguity averse, the ambiguity itself will finally find its way into the security prices in the form of premiums (Liu et al. 2005).

Pricing Techniques for MLS in Incomplete Market

- Pricing Techniques:
  - Arbitrage free pricing method (Cairns et al. 2006b, Bauer et al. 2010).
  - Wang transform (Dowd et al. 2006, Denuit et al. 2007, Lin and Cox 2008, Chen and Cox 2009).
  - Esscher transform (Chen et al. 2010, Li et al. 2010).
  - Instantaneous Sharpe ratio (Young 2008, Bayraktar et al. 2009).
  - Maximum entropy principle (Kogure and Kurachi 2010).
  - Indifference pricing approach (Cui 2008, Cox et al. 2010).
  - Tâtonnement (Economic) pricing approach (Zhou et al. 2011).
- In this study, we focus on the Indifference Pricing and Economic Pricing

Outline

Summary of Our Objectives&Methodologies

- Objective: to explore the effects of risk aversion and ambiguity aversion on mortality risk modeling and pricing
- Main methodologies:
  - Mortality Rate Forecasting Under Parameter Uncertainty
    - Incorporate parameter uncertainty into an asymmetric mortality jump model proposed by Chen et al. (2011)
  - Mortality-linked Security Pricing and Market Equilibrium
    - Economic agent's ambiguity aversion (smooth ambiguity aversion)
    - Market open up (Indifference Pricing)
    - Market equilibrium (Economic Pricing)

Outline

An Asymmetric Mortality Jump Model

- Lee-Carter Model

$$ln(m_{x,t}) = a_x + b_x k_t + e_{x,t}$$

- An Asymmetric Mortality Jump Model proposed by Chen et al.(2011)
  - Negative and positive jumps feature different frequency and severity
  - Mortality jumps have asymmetric time impact on mortality dynamics

$$\begin{cases} \tilde{k}_{t+1} = & \tilde{k}_t + (u - \Lambda) + \sigma Z_{t+1} + Y_{t+1} 1_{\{Y_{t+1}<0\}} 1_{\{N_{t+1}=1\}} \\ k_{t+1} = & \tilde{k}_{t+1} + Y_{t+1} 1_{\{Y_{t+1}<0\}} 1_{\{N_{t+1}=1\}} \end{cases}$$

An Asymmetric Mortality Jump Model(2)

Using the U.S. mortality data from 1900 to 2006, Chen et al. (2011) estimate the parameters.

- The estimate of the probability of positive jumps is equal to one
- This model provides the best fit compared other mortality jump models based on AIC and BIC.

Table 1: Parameter Estimates for the Asymmetric Double Exponential Jump Model

	Parameter Estimate	Standard Error
μ	-0.2457	0.0394
σ	0.3578	0.0390
λ	0.0837	0.0667
η_u	1.4209	0.8654
η_d	N/A	N/A
p	1	N/A

Source: Chen et al. (2011)

Incorporating Parameter Uncertainty in Asymmetric Mortality Jump Model

- We explore the parameter uncertainty on the mean rate of mortality change, μ, in four scenarios

Scenario	Parameter Uncertainty	Ambiguity Averse	Distribution of μ
1	No	No	Use the estimated $\hat{\mu}$ as the true value for forecasting
2	Yes	No	A uniform distribution over 95% confidence interval of $\hat{\mu}$
3	Yes	No	Update the uniform prior via Metropolis-Hasting method
4	Yes	Yes	A uniform distribution over 95% confidence interval of $\hat{\mu}$

Outline

Smooth Ambiguity Preferences

- Smooth ambiguity preference is axiomatized by Klibanoff, et al. (2005).
- It starts from computing the (first order) expected utility conditional on a given prior distribution, then moves to the (second order) expectation of the distorted expected utility over the mass of all priors.
- It allows a separation between ambiguity (belief in regard to uncertainty) and ambiguity aversion (taste with respect to ambiguity).
- Klibanoff et al. (2005) also show that the maxmin preference model is a limiting case of the smooth ambiguity preference model when the degree of ambiguity goes to infinity.

Smooth Ambiguity Preferences(2)

- The ambiguity of the uncertain parameter μ is characterized by a set of priors Λ.
  - Each $\mu \in \Lambda$ describes a possible scenario.
  - $p(\mu)$ is the probabilistic belief over the different scenarios
- The *ex ante* welfare of the agent is measured by

$$V = \phi^{-1}\left(\int_{\Lambda_t} \phi\left(E^{\mu}\left[u(z)\right] p(\mu)\right) d\mu\right)$$

- Following Gollier (2010) , we use an exponential-power specification for (u, ϕ),e.g. each agent has a *negative exponential utility function* and exhibits *constant ambiguity aversion*

$$u(z) = -e^{-\rho z}, \phi(u) = -\frac{(-u)^{1+\gamma}}{1+\gamma}$$

Outline

Indifference Pricing and Market Open-Up

- We adopt indifference pricing to study the range of possible prices$[P^-, P^+]$ for the market to open up.
- The minimal ask price (P^-) for insurer is given by

$$P^- \triangleq \underset{P}{argmax} \left\{ V_{av}^A(P) = \bar{V}_{av}^A \right\}$$

$$V_{av}^A(P) = \underset{\theta^A}{arg\,max} \int_{\Lambda_t} \phi \left(E\left[\mu^A(\widetilde{w}_T^A) | \mu \right] p(\mu) \right) d\mu$$

$$\bar{V}_{av}^A = \underset{\theta^A}{arg\,max} \int_{\Lambda_t} \phi \left(E\left[\mu^A(w_T^A) | \mu \right] p(\mu) \right) d\mu$$

- The maximal bid price (P^+) for investor is given by

$$P^- \triangleq \underset{P}{argmax} \left\{ V_{av}^B(P) = \bar{V}_{av}^B \right\}$$

$$V_{av}^B(P) = \underset{\theta^B}{arg\,max} \int_{\Lambda_t} \phi \left(E\left[\mu^B(\widetilde{w}_T^B) | \mu \right] p(\mu) \right) d\mu$$

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Indifference Pricing and Market Open-Up(2)

Assumptions of agent's wealth process

- Both agents can only invest in either the mortality-linked security or a bank account.
- The mortality-linked security's payoff($g_t(Q_t)$) and insurer's liability($f_t(Q_t)$) are determined by mortality path $Q_t = (q_1, ..., q_t)$.
- There is no borrowing restrictions on both agents.

Proposition 1

With the exponential-power specification, the price range $[P^-, P^+]$ does not depend on the initial wealth of both agents.

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Economic Pricing and Market Equilibrium

- Our economic pricing algorithm can be summarized as
  - Suppose an imaginary auctioneer calls an arbitrary price, say P_0.
  - Given price, agent A and B then decide their supply θ^A and demand θ^B of the mortality-linked security to maximize their end-of-period expected utility, respectively.
  - If the market is not cleared, the auctioneer has to adjust the price until $\theta^A(P) = \theta^B(P)$.

Economic Pricing Algorithm(2)

Setting of optimization problems

- Insurer

$$\theta^A(P) = \underset{\theta^A}{argmax} E\left[\phi\left(E^\mu\left[u\left(\widetilde{w}_T^A\right)\right]\right)\right]$$

$$s.t. \begin{cases} \theta^A \geq 0 \\ \underset{\theta^A}{argmax} E\left[\phi\left(E^\mu\left[u\left(\widetilde{w}_T^A\right)\right]\right)\right] > E\left[\phi\left(E^\mu\left[u\left(w_T^A\right)\right]\right)\right] \quad if\ \theta^A > 0 \end{cases}$$

- Investor

$$\theta^B(P) = \underset{\theta^B}{argmax} E\left[\phi\left(E^\mu\left[u\left(\widetilde{w}_T^B\right)\right]\right)\right]$$

$$s.t. \begin{cases} \theta^B \geq 0 \\ \underset{\theta^B}{argmax} E\left[\phi\left(E^\mu\left[u\left(\widetilde{w}_T^B\right)\right]\right)\right] > E\left[\phi\left(E^\mu\left[u\left(w_T^B\right)\right]\right)\right] \quad if\ \theta^B > 0 \end{cases}$$

Economic Pricing Algorithm (3)

Assumptions of wealth process

- We keep the same assumptions for agents' wealth distribution and the payoffs of the mortality-linked security.
- Particularly,the auctioneer is assumed to adjust the price by following formula:

$$P_{k+1} = P_k + h|P_k|(\theta^B - \theta^A) \qquad h \in R^+$$

Proposition 2

With the exponential-power specification, the equilibrium price P does not depend on the initial wealth of both agents.

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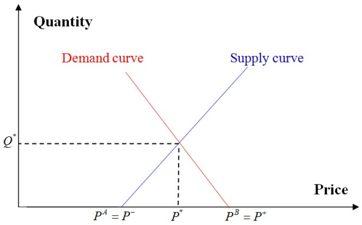
Connection Between Indifference Pricing and Economic Pricing



Demand/Supply Curve From Economic Pricing

Proposition 3

The indifference pricing approach and the economic pricing approach are connected in the sense that $P^- = P^A, P^+ = P^B$.

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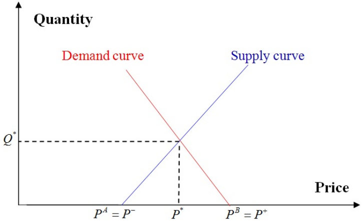

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Settings of Numerical Example

- Agent A(Insurer) has life insurance liabilities $f_t(Q_t) = 500q_t$ at time t,contingent on a mortality index $q_t = m_{65+t,t}$.
- The mortality bond that can be issued by agent A has a similar structure as the first pure mortality bond issued by Swiss Re in December 2003
    - Face Value: \$1 dollar
    - Term: Three years
    - Annual coupon rate:150 basis point+ risk free interest rate (3%)
    - Principle repayment at maturity depends on the $q_t$over the term of the bond

      Principle Repayment $= max\left\{1 - \sum_{t=1}^{3} loss_t, 0\right\}$

      $loss_t = \frac{max(q_t - 1.1q_0, 0) - max(q_t - 1.3q_0, 0)}{0.2q_0}$
- We also assume that there is no trading of the mortality-linked security once it is issued.There are no borrowing constraints for both agents.

Pricing Results for Scenario 1-4

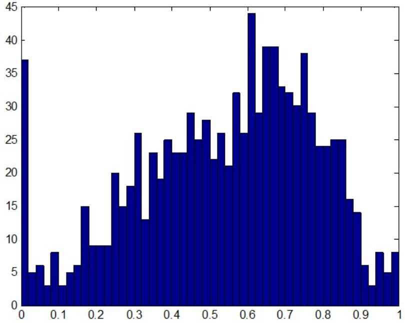
	$E(\mu)$	P^-	P^+	P^*	Q^*	Agent B's Annualized Return (%)	Excess Return (%)
1. Best Estimate	-0.2457	0.5033	0.6244	0.5851	1.4776	5.16%	2.16%
2. Uniform Prior	-0.2457	0.5029	0.6243	0.5849	1.4759	5.17%	2.17%
3. Uniform Prior MH Sampling	-0.2458	0.5033	0.6247	0.5854	1.4809	5.15%	2.15%
4. Uniform prior Ambiguity Aversion	-0.2457	0.5017	0.6243	0.5845	1.4786	5.20%	2.20%

Parameter setting for different scenarios:

Scenario 1-3: $\rho^A = 1, \rho^B = 0.5, \gamma^A = 0, \gamma^B = 0$; Scenario 4: $\rho^A = 1, \rho^B = 0.5, \gamma^A = 5, \gamma^B = 5$

- In scenario 1, there is no parameter uncertainty
- In scenario 2-3, there are parameter uncertainties but both agents are ambiguity neutral
- In scenario 4, there is parameter uncertainty and both agents exhibit ambiguity aversion

Discussion on Scenario 1



- The mortality bond is sold at $ 0.5851, or nearly 41% below its face value ($ 1).
- This is due to the bond's principle payment upon its maturity.
- The mean principle repayment ratio is 0.5408.
- Investor's annualized return is 5.16%
- Investor's excess return is 2.16%.

Effect of Risk Aversion

ρ^A	ρ^B	P^-	P^+	P^*	Q^*	Agent B's Annualized Return	Risk Premium
1.0	0.5	0.5033	0.6244	0.5851	1.4776	5.16%	2.16%
1.0	0.7	0.5033	0.6244	0.5756	1.2999	5.71%	2.71%
0.8	0.5	0.5280	0.6244	0.5881	1.3671	4.99%	1.99%
0.8	0.7	0.5280	0.6244	0.5802	1.1818	5.44%	2.44%

Proposition 4

Using an exponential utility, the risk aversion of agent B does not affect the maximal bid price(P^+).

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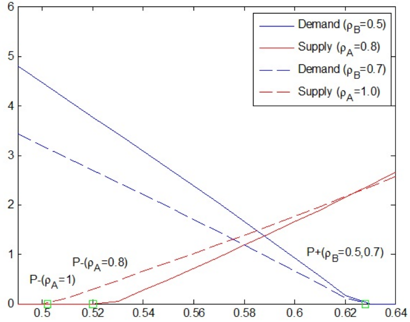
Risk Aversion & Market Equilibrium



- As insure becomes *less* risk averse
  - Supply curve shifts downward
  - The minimal ask price increases
  - $P^* \uparrow \; Q^* \downarrow$
- As investor becomes *more* risk averse
  - Demand curve rotates counter clockwise
  - The maximal bid price remain the same
  - $P^* \downarrow \; Q^* \downarrow$

Effect of Ambiguity Aversion

γ^A	γ^B	P^-	P^+	P^*	Q^*	Agent B's Annualized Return	Excess Return	Ambiguity Premium
0	0	0.5030	0.6243	0.5850	1.4780	5.17%	2.17%	0.01%
0	5	0.5030	0.6243	0.5847	1.4726	5.19%	2.19%	0.03%
0	10	0.5030	0.6243	0.5844	1.4672	5.20%	2.20%	0.04%
5	0	0.5017	0.6243	0.5848	1.4840	5.18%	2.18%	0.02%
5	5	0.5017	0.6243	0.5845	1.4786	5.20%	2.20%	0.04%
5	10	0.5017	0.6243	0.5842	1.4732	5.21%	2.21%	0.06%
10	0	0.5003	0.6243	0.5846	1.4898	5.19%	2.19%	0.03%
10	5	0.5003	0.6243	0.5843	1.4844	5.21%	2.21%	0.05%
10	10	0.5003	0.6243	0.5841	1.4791	5.22%	2.22%	0.06%

Proposition 5

In an exponential-power specification, the maximal bid price(P^+) is not affected by either risk aversion or ambiguity aversion of agent B.

Effect of Ambiguity Aversion

γ^A	γ^B	P^-	P^+	P^*	Q^*	Agent B's Annualized Return	Excess Return	Ambiguity Premium
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Ambiguity Aversion & Market Equilibrium

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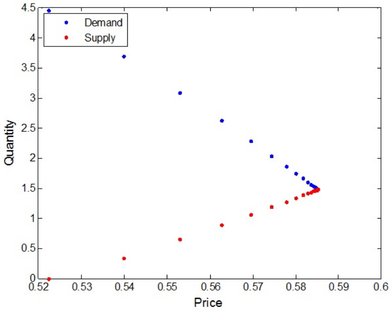
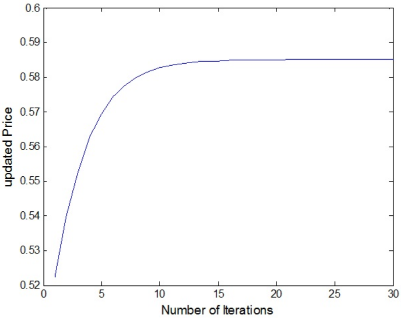
- The ambiguity aversion has similar effects on market equilibrium.
- Ambiguity aversion has a much smaller effect than risk aversion.

Performances of Economic Pricing Algorithm





Summary

- We find that indifference pricing and economic pricing are intrinsically connected.
- We find that changes in risk aversion and ambiguity aversion have similar effects on the price range and the equilibrium price/quantity. However, risk aversion plays a more prominent role in our numerical example.
- Future research
    - Relax the assumption of a competitive and information efficient market for mortality-linked securities
    - Relax the assumption of no secondary market for mortality-linked securities and thus no trade after the first issuance

Discussion

- Questions/Suggestions?