Yaari's LifeCycle Model in the 21st Century: Consumption Under a Stochastic Force of Mortality

(Joint work with H. Huang and T.S. Salisbury)

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- His model is at the foundation of much of modern micro-economics.
- The Yaari (1965) model was based on a deterministic force of mortality in which the entire survival curve is known at time zero.
- In this paper we extend the Yaari (1965) model – with no annuities – to a world with stochastic mortality rates.

The LifeCycle Model is Used to Provide Investment Advice

"...As far as I am aware, no one has challenged the view that if people were capable of it, they ought to plan their consumption, saving and retirement according to the principles enunciated by Modigliani and Brumberg in 1950s..."

Professor Angus S. Deaton, Princeton University, 2005

Force of Mortality

- Let $\lambda(t)$ denote the mortality rate of a cohort of a population. Let $\mathcal{F}_t = \sigma\{\lambda(q) \mid q \leq t\}$ be the filtration determined by λ. Then individuals in the population have lifetimes of length ζ satisfying

$$P(\zeta > s \mid \zeta > t, \mathcal{F}_\infty) = e^{-\int_t^s \lambda(q)\,dq}. \tag{1}$$

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- Assume further that $\lambda(t)$ is a Markov process, and define the survival function $p(t, s, \lambda)$ by

$$p(t, s, \lambda) = E\left[e^{-\int_t^s \lambda(q)\,dq} \mid \lambda(t) = \lambda\right]. \tag{2}$$

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$$p(t, s, \lambda) = E\left[e^{-\int_t^s \lambda(q)\,dq} \mid \lambda(t) = \lambda\right]. \qquad (2)$$

- This gives the conditional probability of surviving from time t to time s, given knowledge of the mortality rate at time t. Therefore

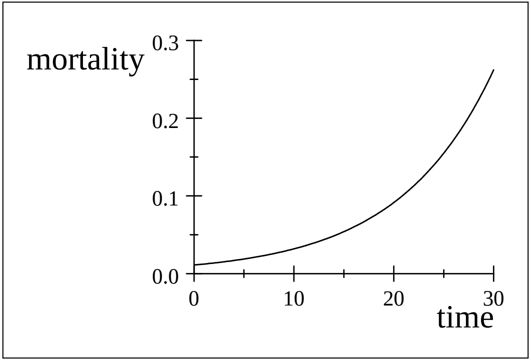
$$P(\zeta > s \mid \zeta > t, \mathcal{F}_t) = E\left[e^{-\int_t^s \lambda(q)\,dq} \mid \mathcal{F}_t\right] = p(t, s, \lambda(t)). \qquad (3)$$

If $t = 0$ then we write $p(s, \lambda)$ for $p(0, s, \lambda)$.

Gompertz Mortality

A very popular law of mortality is the Gompertz law of mortality.

$$\lambda(t) = \frac{1}{b} \exp\left(\frac{x + t - m}{b}\right),$$



Notes: $x = 65$, $m = 89.3$, $b = 9.5$ and $p(0, 35, 0.0081) = 5\%$

The Yaari (1965) LifeCycle Model

- Objective Function:

$$J = \max_{c} E\left[\int_0^D e^{-\rho t} u(c(t)) 1_{\{t \leq \zeta\}} dt\right],$$

where $\zeta \leq D$ is the remaining lifetime satisfying $\Pr[\zeta > t] = p(t, \lambda_0)$.

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- When $\lambda(t)$ is deterministic, the optimal consumption $c^*(t)$ and $1_{\{t \le \zeta\}}$ are independent, so by Fubini's theorem:

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$$F_t(t) = v(t, F(t))F(t) + \pi_0 - c(t),$$

with $F(0) = W > 0$ and $F(D) = 0$.

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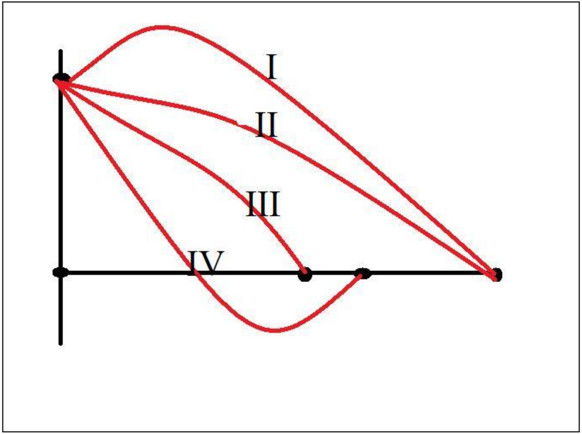
- The investment return $v = v(t, F)$ is defined by:

$$v(t, F) = \begin{cases} r + \xi\lambda(t), & F \geq 0, \\ R + \lambda(t), & F < 0, \end{cases}$$

Graphical View of the Solution



Four Different Wealth Trajectories

Solution to LCM with DfM

- When $v(t) = r$, and $u(c) = c^{(1-\gamma)}/(1-\gamma)$ then by the Euler-Lagrange Theorem, the optimal $F(t)$ must satisfy a second-order non-homogenous differential equation in regions where $F(t) \neq 0$.

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- The PDE to solve is:

$$F_{tt}(t) - \left(\frac{r-\rho-\lambda(t)}{\gamma} + r\right) F_t(t) + r\left(\frac{r-\rho-\lambda(t)}{\gamma}\right) F(t)$$

$$= \quad -\left(\frac{r-\rho-\lambda(t)}{\gamma}\right)\pi_0.$$

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- Related literature: Leung (1990), Davies (1981), Lachance (2010).

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- Optimal trajectory of wealth is:

$$\begin{aligned} F(t) &= \left(F(0) - c^*(0)\int_0^t e^{ks}(p(s,\lambda_0))^{1/\gamma}e^{-rs}ds\right)e^{rt} \\ &= \left(F(0) - c^*(0)a_x^t(r-k, m^*, b)\right)e^{rt} \end{aligned}$$

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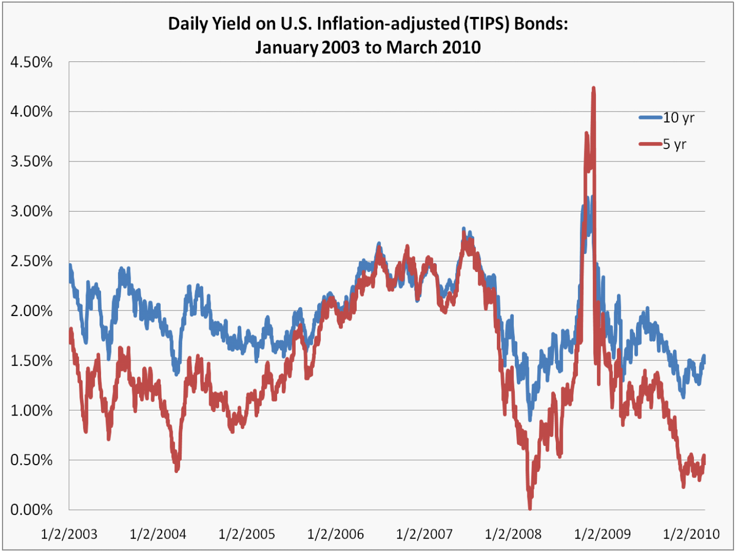
- Initial consumption rate is...

$$c^*(0) = \frac{F(0)}{a_x^D(r-k,m^*,b)},$$

where $k = (r-\rho)/\gamma$ and $m^* = m + b\ln[\gamma]$.

Calibration: What Interest Rate Should we Use?



Numerical Results (DfM) #1

Optimal Consumption Rate				
Coefficient of Relative Risk Aversion (CRRA) $\gamma = 4$				
Nest Egg of $100 Invested at Following REAL Rates...				
	$r = 0.5\%$	$r = 1.5\%$	$r = 2.5\%$	$r = 3.5\%$
Age 65				
5 Years Later				
10 Years Later				
20 Years Later				
30 Years Later				

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	$r = 0.5\%$	$r = 1.5\%$	$r = 2.5\%$	$r = 3.5\%$
Age 65	$3.330	$3.941	$4.605	$5.318
5 Years Later	$3.286	$3.888	$4.544	$5.247
10 Years Later	$3.212	$3.801	$4.442	$5.130
20 Years Later	$2.898	$3.429	$4.007	$4.627
30 Years Later	$2.156	$2.552	$2.982	$3.444

Numerical Results (DfM) #2

Optimal Initial Withdrawal Rate (IWR) from \$100				
As a Function of Pension Income π_0				
Depending on the Coefficient of Relative Risk Aversion				
	$\gamma = 1$	$\gamma = 2$	$\gamma = 4$	$\gamma = 8$
No Pension	6.330%	5.301%	4.605%	4.121%
$\pi_0 = \$1$				
$\pi_0 = \$2$				
$\pi_0 = \$5$				
Note: Gompertz Mortality ($m = 89.3$, $b = 9.5$) and $r = 2.5\%$				

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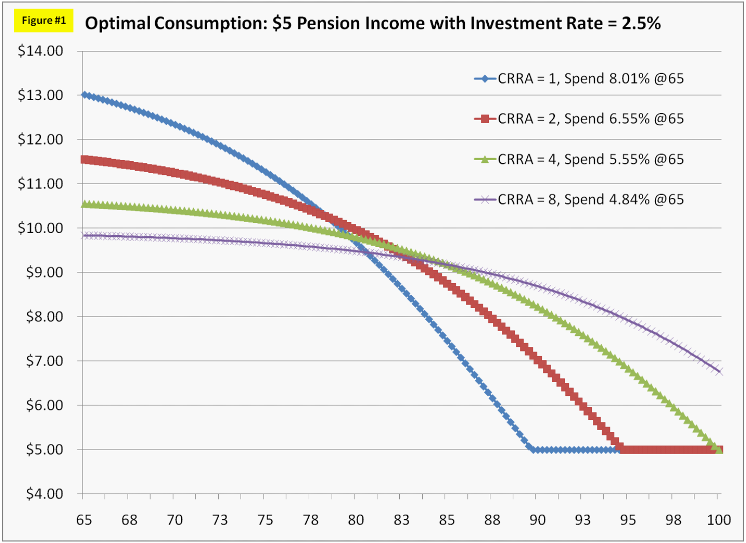
No Pension	6.330%	5.301%	4.605%	4.121%
$\pi_0 = \$1$	6.798%	5.653%	4.873%	4.324%
$\boldsymbol{\pi}_0 = \$2$	7.162%	5.924%	5.078%	4.480%
$\boldsymbol{\pi}_0 = \$5$	8.015%	6.553%	5.551%	4.839%
Note: Gompertz Mortality $(m = 89.3,\ b = 9.5)$ and $r = 2.5\%$				

Retire with a $100 Nest Egg and a $5 per year pension...



Good advice, bad reason?

The New York Times

Sunday, September 25, 2005

Commercial Real Estate, Pages 22-25

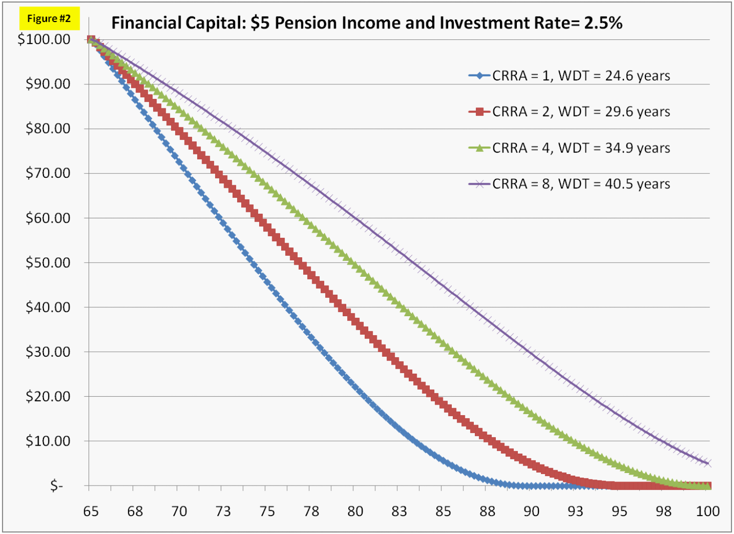
SundayBusiness

Section 3

New Advice to Retirees: Spend More at First, Cut Back Later

Two planners challenge the traditional approach for managing that nest egg.

Wealth Trajectory



Numerical Results (DfM) #3

How Does Pensionization Impact Consumption?				
Percent	**Risk Aversion** $\gamma = 4$		**Risk Aversion** $\gamma = 8$	
Pensionized	**Age 65**	**Age 80**	**Age 65**	**Age 80**
0%				
20%				
40%				
60%				
100%				

Numerical Results (DfM) #3

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Pensionized	**Age 65**	**Age 80**	**Age 65**	**Age 80**
0%	\$4.605	\$4.007	\$4.121	\$3.844
20%	\$5.263	\$4.580	\$4.801	\$4.478
40%	\$5.795	\$5.042	\$5.385	\$5.024
60%	\$6.227	\$5.419	\$5.937	\$5.538
100%	\$6.330	\$6.330	\$6.330	\$6.330

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- How does optimal consumption behavior change and what is the impact of "longevity risk aversion" on the optimal plan?

Deterministic force of Mortality (DfM) World

$$ {}_{10}p_{90} = \frac{{}_{35}p_{65}}{{}_{25}p_{65}} $$

Conditional Survival Probability:						
	$x = 65$	$x = 75$	$x = 85$	$x = 90$	$x = 95$	$x = 100$
To 65	1.000					
To 75	0.8659	1.000				
To 85	0.5733	0.6620	1.000			
To 90	**0.3696**	0.4268	0.6447	1.000		
To 95	0.1758	0.2031	0.3067	0.4757	1.000	
To 100	**0.0500**	0.0577	0.0872	**0.1353**	0.2844	1.000
λ_x	0.0081	0.0232	0.0667	0.1129	0.1911	0.3234

Stochastic force of Mortality (SfM) World

Conditional Survival Probability:						
	$x = 65$	$x = 75$	$x = 85$	$x = 90$	$x = 95$	$x = 100$
To 65	1.000					
To 75	0.8659	1.000				
To 85	0.5733	**?**	1.000			
To 90	0.3696	**?**	**?**	1.000		
To 95	0.1758	**?**	**?**	**?**	1.000	
To 100	0.0500	**?**	**?**	**?**	**?**	1.000
λ_x	0.0081	**?**	**?**	**?**	**?**	**?**

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$$\max_{c(s) \text{ adapted}} E\left[\int_t^T e^{-\int_t^s (r+\lambda(q))\, dq} u(c(s)) ds \,\middle|\, \lambda(t) = \lambda, F(t) = F \right]$$

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- We condition on wealth $F(t)$ and the mortality rate $\lambda(t)$.
- The wealth process (budget constraint) still satisfies $dF(t) = (rF(t) - c(t))dt$.

Solution: Optimal Consumption under SfM

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$$e^{-\int_0^t (r+\lambda(q))\,dq} J(t, \lambda(t), F(t)) + \int_0^t e^{-\int_t^s (r+\lambda(q))\,dq} u(c(s))\,ds.$$

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- It can be written as:

$$e^{-\int_0^t (r+\lambda(q))\,dq} J(t,\lambda(t),F(t)) + \int_0^t e^{-\int_t^s (r+\lambda(q))\,dq} u(c(s))\,ds.$$

- By Ito's lemma, we have the following HJB equation for the value function:

$$\sup_c \{u(c) - cJ_F\} + J_t - (r+\lambda)J + rFJ_F + \mu(t)\lambda J_\lambda + \frac{\sigma^2\lambda^2}{2} J_{\lambda\lambda} = 0$$

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- Apply the 1st order condition $c^* = J_F^{-\frac{1}{\gamma}}$. We obtain

$$c^* = Fa^{-\frac{1}{\gamma}}$$

and get the following equation for $a(t,\lambda)$:

$$a_t - (r\gamma + \lambda)a + \gamma a^{1-\frac{1}{\gamma}} + \mu(t)\lambda a_\lambda + \frac{\sigma^2\lambda^2}{2} a_{\lambda\lambda} = 0$$

with boundary condition $a(T,\lambda) = 0$.

Numerical Results (SfM)

Main Question: How does the volatility of mortality (σ), impact the optimal initial withdrawal rate? The drift $\mu(t)$ of the mortality rate process is calibrated to fit a Gompertz survival curve $(m = 89.3, b = 9.5)$, such that $p(35, 0.0081) = 5\%$

Optimal Initial Withdrawal Rate (IWR)						
Volatility	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
$\sigma = 0$						
$\sigma = 15\%$						
$\sigma = 25\%$						
Notes: Retirement age 65, interest rate $r = 2\%$, mortality $\lambda_0 = 0.0081$						

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Volatility	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
$\sigma = 0$	7.59%	6.12%	5.58%	5.02%	4.78%	4.61%
$\sigma = 15\%$	7.52%	6.12%	5.60%	5.04%	4.80%	4.62%
$\sigma = 25\%$	7.44%	6.12%	5.62%	5.06%	4.82%	4.63%
Notes: Retirement age 65, interest rate $r = 2\%$, mortality $\lambda_0 = 0.0081$						

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- Denote by $c^{\text{SfM}}(t, \lambda, F)$ the optimal consumption at time t, given $\lambda(t) = \lambda$ and $F(t) = F$, under a stochastic force of mortality (SfM) model. Denote by $c^{\text{DfM}}(t, F)$ the optimal consumption at time t, when $F(t) = F$, under a deterministic force of mortality (DfM) model.

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- **THEOREM**: *Assume that the survival functions for the two models agree:* $p^{\mathrm{SfM}}(t, \lambda_0) = p^{\mathrm{DfM}}(t, \lambda_0)$ *for every* $t \geq 0$*, and that utility is CRRA(*γ*). There are three regimes: (a)*
$\gamma > 1 \Longrightarrow c^{\mathrm{SfM}}(0, \lambda_0, F) \geq c^{\mathrm{DfM}}(0, F)$. *(b)*
$\gamma = 1 \Longrightarrow c^{\mathrm{SfM}}(0, \lambda_0, F) = c^{\mathrm{DfM}}(0, F)$. *(c)*
$0 < \gamma < 1 \Longrightarrow c^{\mathrm{SfM}}(0, \lambda_0, F) \leq c^{\mathrm{DfM}}(0, F)$.

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- You might have noticed that the optimal IWR was invariant to mortality volatility σ when $\gamma = 1$ (which is logarithmic utility). This is not a coincidence. It is a theorem.
- Denote by $c^{\text{SfM}}(t, \lambda, F)$ the optimal consumption at time t, given $\lambda(t) = \lambda$ and $F(t) = F$, under a stochastic force of mortality (SfM) model. Denote by $c^{\text{DfM}}(t, F)$ the optimal consumption at time t, when $F(t) = F$, under a deterministic force of mortality (DfM) model.
- **THEOREM**: *Assume that the survival functions for the two models agree:* $p^{\text{SfM}}(t, \lambda_0) = p^{\text{DfM}}(t, \lambda_0)$ *for every* $t \geq 0$*, and that utility is CRRA(*γ*). There are three regimes: (a)*
$\gamma > 1 \Longrightarrow c^{\text{SfM}}(0, \lambda_0, F) \geq c^{\text{DfM}}(0, F)$. *(b)*
$\gamma = 1 \Longrightarrow c^{\text{SfM}}(0, \lambda_0, F) = c^{\text{DfM}}(0, F)$. *(c)*
$0 < \gamma < 1 \Longrightarrow c^{\text{SfM}}(0, \lambda_0, F) \leq c^{\text{DfM}}(0, F)$.
- Proof in the paper...

Conclusion

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- Any horse race (i.e. comparison) between deterministic and stochastic mortality models, should ensure *rational mortality expectations.*
- Future research will examine the impact of annuities in such a model.