A time-dependent age-factor extension to the Lee-Carter model

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Introduction

Outline of presentation

- Introduction
- Time dependency
- Linear model
- P-Splines
- Model
- Conclusions

Introduction

- Lee Carter (1992) model:

$$\log(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}$$
$$k_t = k_{t-1} + c + e_t$$

- a_x: general level of mortality;
- b_x: relative age reduction in mortality rates;
- k_t: general level reduction in mortality rates;

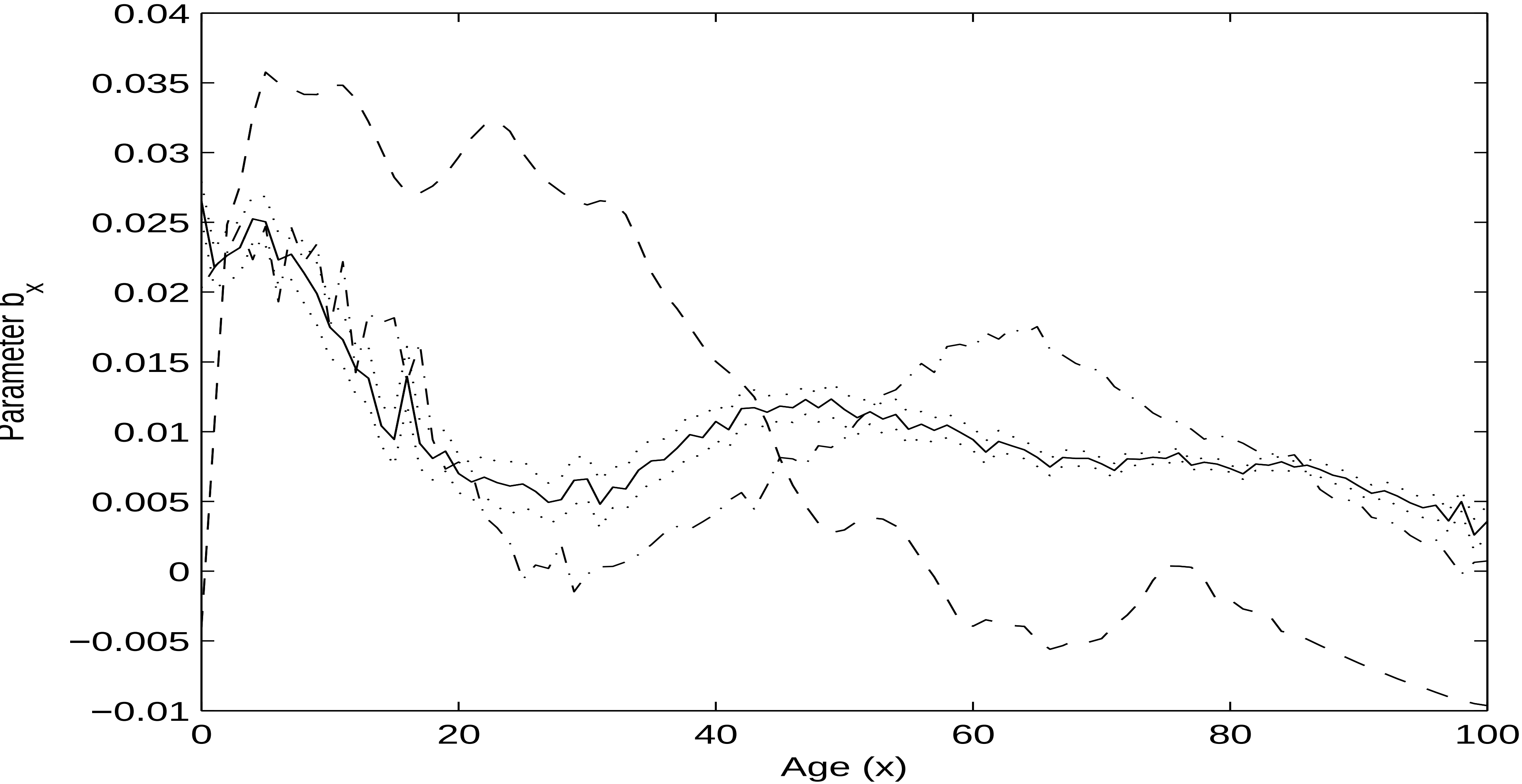
Lee-Carter

- Assumption: b_x is time-independent.
- Empirical evidence this assumption is violated:
  - E.g. Booth et al. (2002), solution: use short time horizon;
  - Scientists tend to underestimate life expectancy increases;
  - Anecdotal evidence that focus of medicine towards older ages over time.
- Why do we use it?
- Makes life easy.

Solution

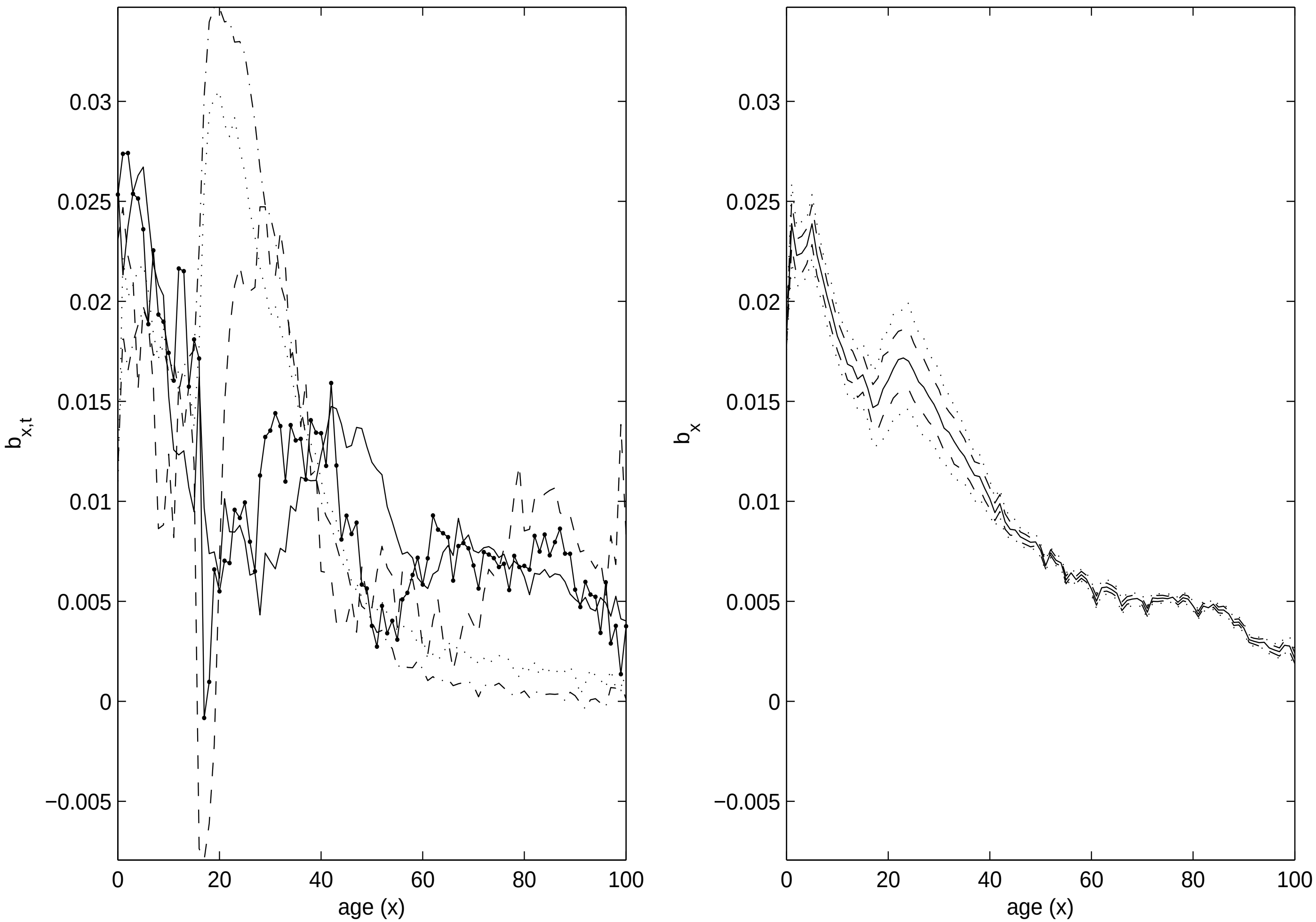
- Find a way to estimate $b_{x,t}$ and k_t using parametric form.
- Find a way to forecast both $b_{x,t}$ and k_t (i.e., using the parametric form).
- Application (often used):
  - English & Wales mortality data;
  - Males;
  - Ages $0, 1, \ldots, 100^{+}$.
- Lee-Carter model without cohort effect.

Model



Solid curve: best estimate, dotted curve: 95% confidence bounds;
Dashed curve: 1932-1950; Dashed-dotted: 1987-2006.

Estimated age parameter



- Left panel: time dependent.
- Right panel: classical LC estimates with 95% CI.
- Clear evidence of change in pattern;
- Seems older ages get higher b_x over time;
- Need to take it into account when forecasting.

Dotted: 1960, dashed-dotted: 1970, dashed: 1980; solid-dotted: 1990; solid: 1996.

Time dependent age parameter

- Identification issue: $b_{x,t}$ and k_t would not be uniquely defined.
- Solution: parametric form $b_{x,t}$.
- Question: which parametric form?
- Insights: Estimate $b_{x,t}$ using b_x from a rolling window of 20 years;
- Similar to linear moving average smoothing method.
- Results shown in previous slide.

Time dependent age parameter

- Now we have our estimated values of $b_{x,t}$.
- How to make forecasts?
  - k_t using ARIMA model;
  - $b_{x,t}$ using a parametric form;
- We investigated three methods:
  - Polynomial expansion;
  - P-Splines;
  - Linear age-dependent method.

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Polynomial expansion

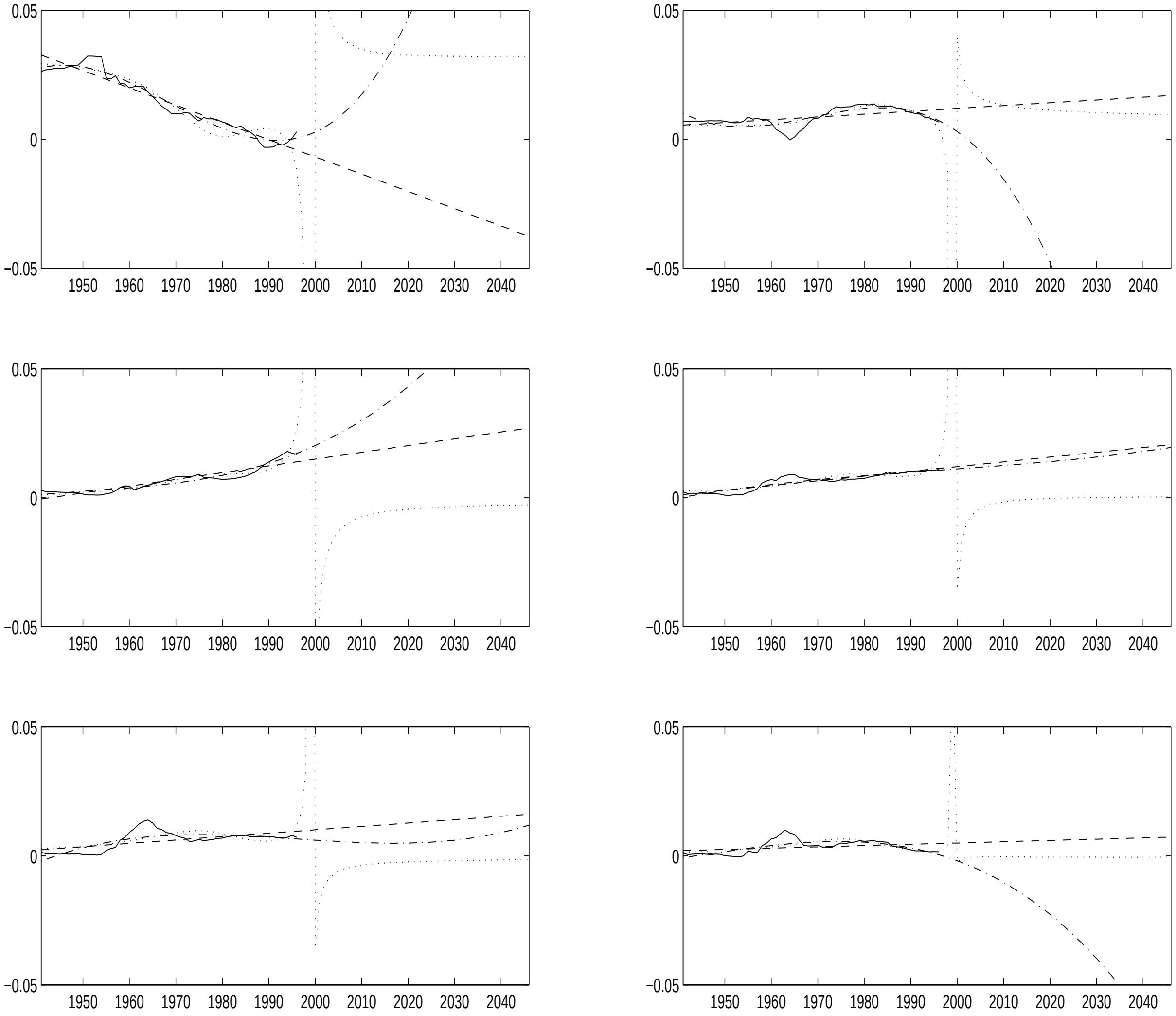
First idea

- Easy way: use polynomial in time.
- Hence:

$$b_{x,t} = c_{0,x} + c_{1,x}t + \ldots$$

- If higher order: $\sum_x b_{x,t} \neq 1$, thus dived all $b_{x,t}$ by $\sum_x b_{x,t}$.
- Problems: realistic forecasts?
- Linear: effect slowing reduces over time?
- Polynomial: large positive/negative values?

Results

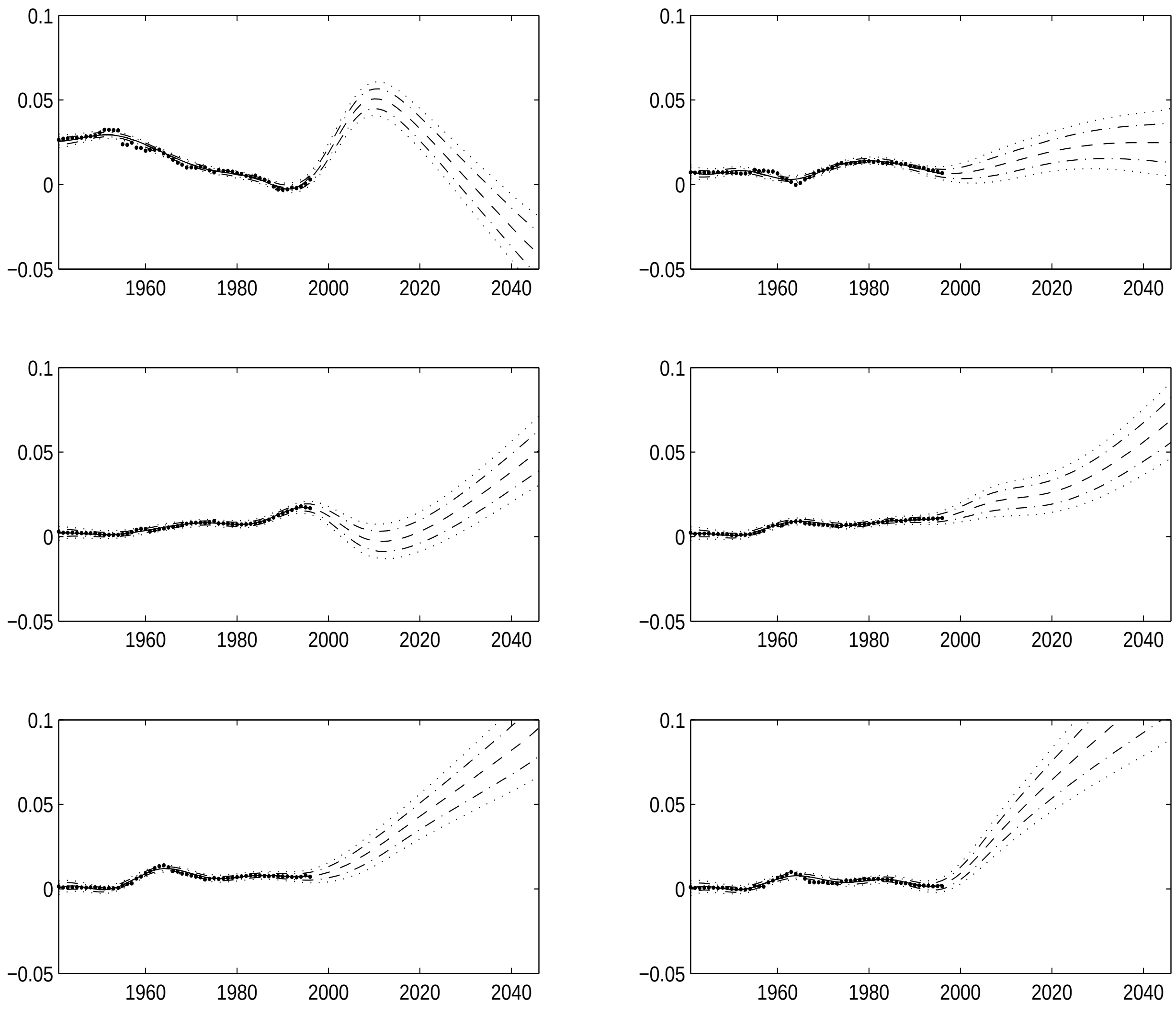


P-Splines

Idea

- More flexible way of smoothing: P-Splines.
- Used in mortality modeling.
- Mixed results: sometimes good estimates, depends strongly on penalty.
- Using penalty, one can obtain forecasts + uncertainty.
- Would it work for $b_{x,t}$?

Results



Linear age-dependent

Idea

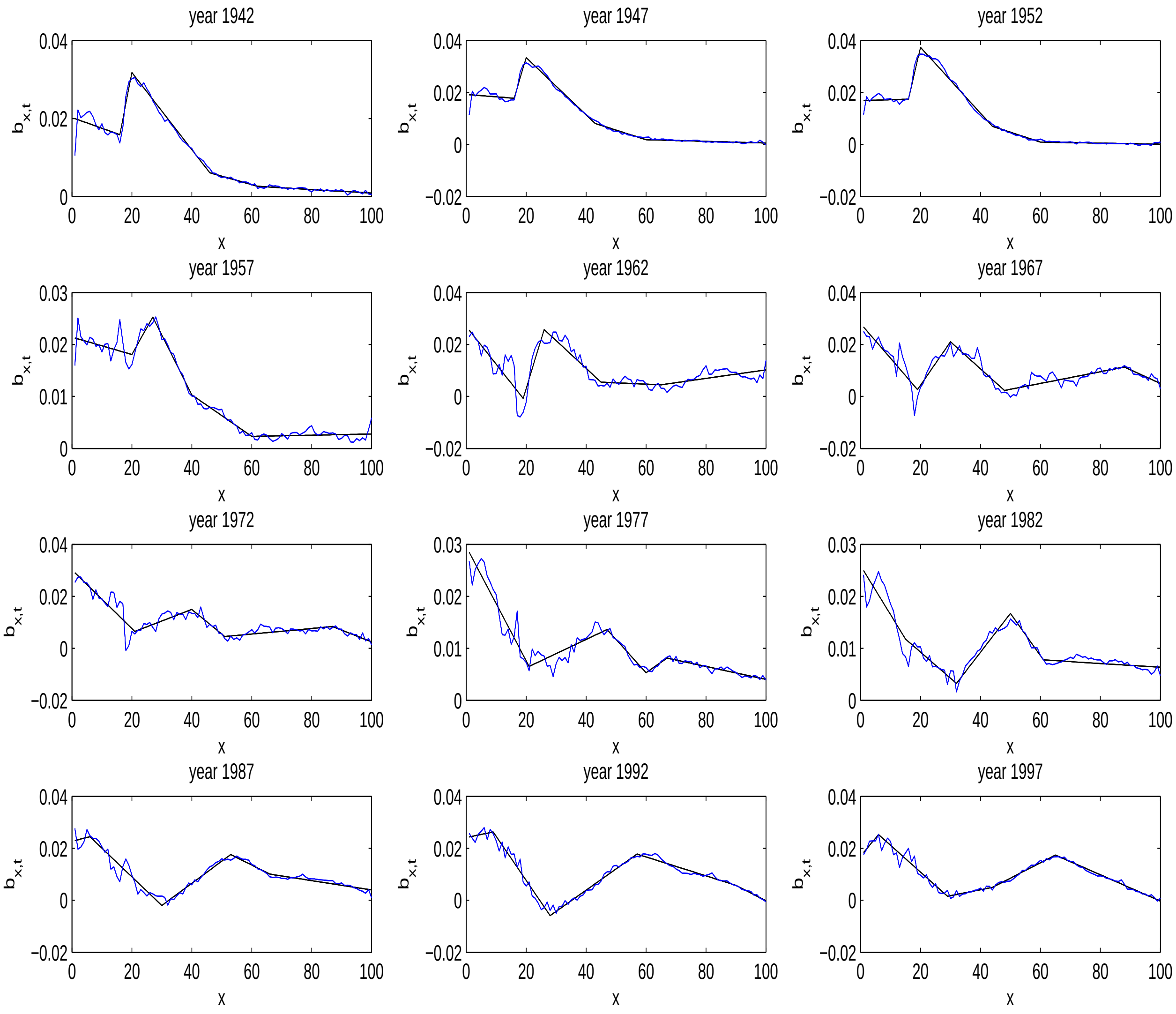
- “Simple” models do not seem to work.
- Why?
- The structure of the decline in mortality probabilities changes over time.
- Stronger reduction tends towards higher ages.
- Example:
  - Focus on young mortality (infectious diseases);
  - Focus on middle age mortality (cancer);
  - Focus on advanced age mortality.

Model 1

- Model for each year the parameter $b_{x,t}$ in a linear way.
- Have 5 different age groups;
- Within each age group change is linear;
- Age groups changes over time;
- Parameter estimates using OLS & iterative method;
- Iterative method: find the optimal age groups.

- Disadvantage: not applicable to forecast.
- Advantage: We can investigate the pattern in $b_{x,t}$.

Results 1


year 1942
year 1947
year 1952
year 1957
year 1962
year 1967
year 1972
year 1977
year 1982
year 1987
year 1992
year 1997
$b_{x,t}$
x
0
20
40
60
80
100
0.04
0.03
0.02
0.01
0
−0.02

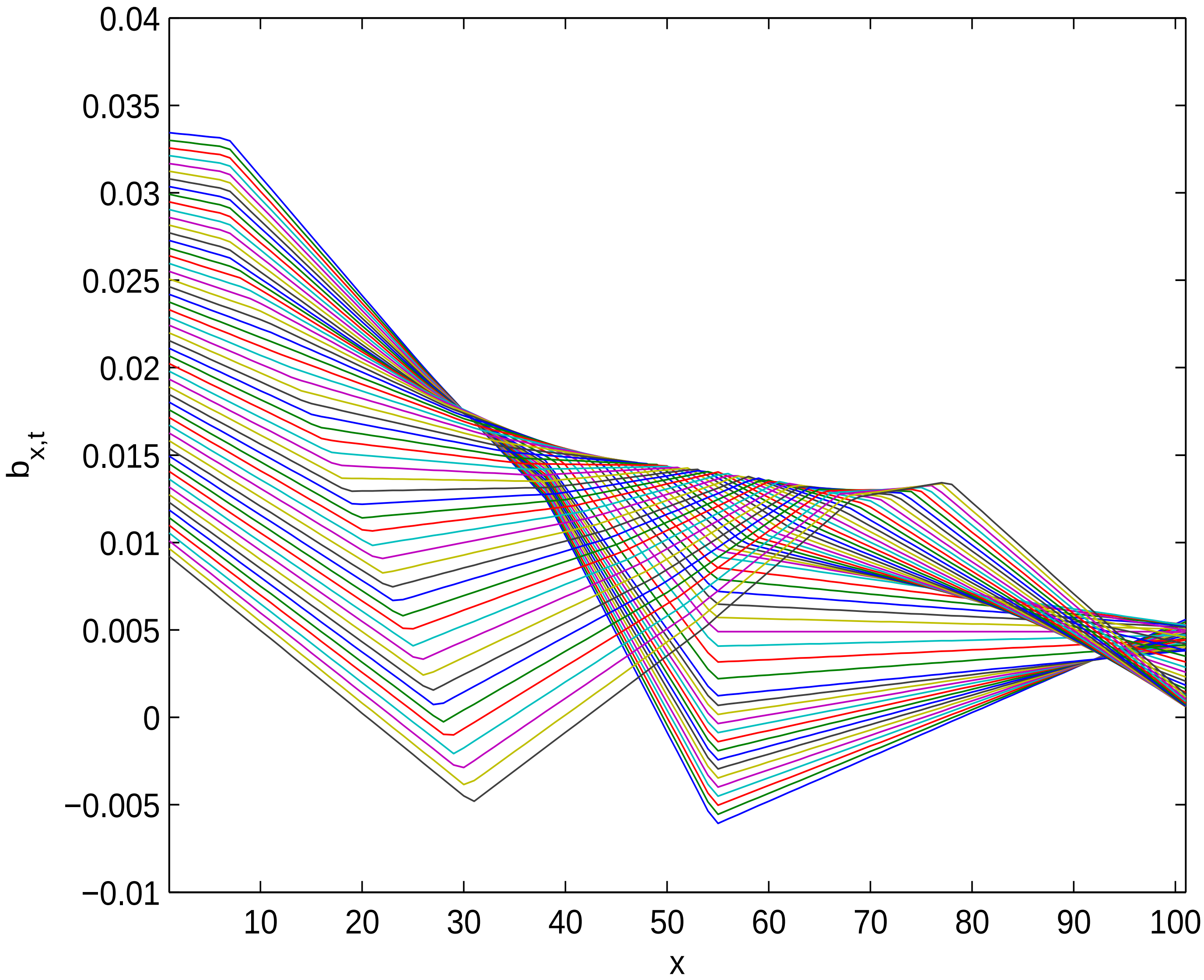
Model 2

- Estimate the parameters in a parametric way:

$$\begin{aligned}\beta_{x,t} =& \gamma_{0,t} + \gamma_{1,t} x + \gamma_{2,t}(x - X_{1,t})_+ + \ldots + \gamma_{5,t}(x - X_{4,t})_+ \\ X_{i,t} =& \quad a_i + b_i(t - c_i)_+, \quad \text{for } i = 1, \ldots, 4; \\ \gamma_{i,t} =& \quad d_i + e_i, \quad \text{for } i = 0, \ldots, 5.\end{aligned}$$

- Parameter estimates using GMM;
  - Many moment conditions;
  - Few parameters;
- Model overidentified ⇒ two step GMM, using Newton-Ralphson method.

Results 2


0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
−0.005
−0.01
$b_{x,t}$
10
20
30
40
50
60
70
80
90
100
x

Conclusions

Conclusions

- Age-time interactions in the Lee-Carter model are present;
- We should take it into account $\Rightarrow$ otherwise underestimate reduction in mortality at advanced ages;
- Age-time interactions hard to estimate;
- Age-time interactions hard to model;
- We came up with a possible model;
- Explanation in higher reduction at more advanced ages is logical;
- Use standard Lee-Carter model with **care**!