– Longevity Seven, Frankfurt –

Coherent Pricing of Life Settlements Under Asymmetric Information

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Background & Literature Review

The life settlement market:

- Abiding investment opportunity
- Senior insureds w/ below average health $\Leftarrow$ "Viatical settlements" (1980s)
- Securitization in the capital market (Chen et al. (2011), Stone and Zissu (2006))
- Limited number of contracts $\Rightarrow$ idiosyncratic risk factors

Recent market investigations:

- Expected returns 8-12% from a policy-by-policy basis (Gatzert (2010))
- Open-end life settlement funds returned $\approx$ 4.8% (Braun et al. (2011))
- Bad quality of underlying life expectancy estimates?
  - Systematic biases – should be swiftly corrected
  - Unsystematic errors – cannot explain aggregate underperformance
- Rating agencies declined rating these "death bonds" due to "unique risks"

Main Findings

Different view points based on adverse selection

- One-period expected utility model $\Rightarrow$ offer price in competitive market
  - With symmetric information on health condition
  - With asymmetric information on health condition
  - Adjustment of pricing scheme $\Leftrightarrow$ clientele effects (Hoy and Polborn (2000), Villeneuve (2003))
- Extended framework $\Rightarrow$ applicable pricing formulas
  - Frailty model $\Rightarrow$ heterogeneity in life tables
  - Life-time utility evaluation $\Rightarrow$ threshold set for settling
  - Generalizations $\Rightarrow$ option to settle in later periods
- Numerical examples
  - Impact of asymmetric information varies
  - Extreme cases: no effect or market breakdown

Symmetric Information

Simple one-period expected utility model

Representative policyholder

- One-period term-life insurance: F
- No future contingent premiums, zero cash surrender value
- Condition: p (survival probability to the end of the period)
- $u(\cdot)$ and $v(\cdot)$: utilities from life insurance benefits

Under a competitive secondary life market:

- $OP^{\text{sym}}(p) = \frac{(1-p)\times F}{1+R}$
  - $U^{\text{r}} = p \times u(0) + (1-p) \times v(F)$
  - $U^{\text{s}} = p \times u((1-p)F) + (1-p) \times v((1-p)F)$
  - Settle if and only if $U^{\text{s}} \geq U^{\text{r}}$

Asymmetric Information

$\bar{p}$: estimate of p from third party $\Rightarrow f(p|\bar{p})$

Without considering policyholder's behavior:

- $OP^{\mathrm{a}}(\bar{p}) = \frac{\mathbb{E}[(1-p)|\bar{p}]\times F}{1+R}$
- Not economically rational!

With considering policyholder's behavior:

- $U^{\mathrm{r}} = p \times u(0) + (1-p) \times v(F)$
- $U^{\mathrm{s}}(p, OP) = p \times u(OP \times (1+R)) + (1-p) \times v(OP \times (1+R))$
- $U^{\mathrm{s}}(p, OP) - U^{\mathrm{r}}(p) \geq 0$
  $\Leftrightarrow p \geq \frac{v(F)-v(OP\times(1+R))}{u(OP\times(1+R))-u(0)+v(F)-v(OP\times(1+R))} \triangleq p^*(OP)$
- $OP^e(\bar{p}) \triangleq \arg\max_x \left\{ \int_{p^*(x)}^{1} ((1-p)F - x(1+R)) f(p|\bar{p}) dp = 0 \right\}$
- Average Clientele Risk?
  - Time point: settling vs. purchasing the policy
  - Derived price: independent settlement vs. level premium

Implication

Proposition

With asymmetric information with respect to p, the rational expectation offer price, $OP^{e}(\bar{p})$, will be smaller than $OP^{a}(\bar{p})$, for all estimates $\bar{p}$.

Proof. It is sufficient to show that

$$\begin{aligned} & \int_{p^*(OP^a)}^{1} ((1-p)F - (1+R) \times OP^a) f(p|\bar{p}) dp \leq 0 \\ \Leftrightarrow \quad & F \times \int_{p^*(OP^a)}^{1} ((1-p) - \mathbb{E}[(1-p)|\bar{p}]) f(p|\bar{p}) dp \leq 0 \\ \Leftrightarrow \quad & \mathbb{E}[p|\bar{p}, p \geq p^*(OP^a)] - \mathbb{E}[p|\bar{p}] \geq 0 \end{aligned}$$

- Value of the policy is far underestimated $\Rightarrow$ reject
- Offer price exceeds intrinsic value $\Rightarrow$ settle
- Explanation for the discrepancy between expected and realized returns!
- Coherent pricing should take policyholder's decision into account

Heterogenous Life Tables

Extended framework:

- Multi-period environment
- Whole-life policy, annual premium P, death benefit F

Heterogeneity w.r.t. individual mortality rates (Vaupel et al. (1979), Hoermann and Russ (2008))

- Current frailty models: fail to connect average of (heterogeneous) individual tables to population table
- $\mathbb{E}^j[{}_\tau p_x{}^j(T)] = {}_\tau p_x(T)$, $\forall \tau$, and ${}_\tau p_x{}^j(T) \in [0,1]$, $\forall \tau, j$
- We propose:

$$ {}_\tau p_x{}^j(T) = {}_\tau p_x(T) + A_j \times \min\{{}_\tau p_x(T), 1 - {}_\tau p_x(T)\} e^{-\gamma(\tau-1)}, $$

s.t. $A_j \in [-1,1]$, and $\mathbb{E}[A_j] = 0$.

Policyholders' Decision Making

Value function when retaining:

$$V_T^{r}(W_0,j)=\max_{c_\tau}\sum_{\tau=1}^{\omega-x}{}_{\tau-1}p_x{}^j(T)\times\beta^{\tau-1}\times u(c_\tau-P)+\sum_{\tau=1}^{\omega-x}({}_{\tau-1}p_x{}^j(T)-{}_{\tau}p_x{}^j(T))\times\beta^{\tau}\times v(W_\tau+F),$$

s.t.

$$W_\tau=(W_{\tau-1}-c_\tau)\times\frac{1}{p(\tau-1,1)},\ \tau=1,\dots,\omega-x.$$

Value function when settling:

$$V_T^{s}(W_0,OP,j)=\max_{c_\tau}\sum_{\tau=1}^{\omega-x}{}_{\tau-1}p_x{}^j(T)\times\beta^{\tau-1}\times u(c_\tau)+\sum_{\tau=1}^{\omega-x}({}_{\tau-1}p_x{}^j(T)-{}_{\tau}p_x{}^j(T))\times\beta^{\tau}\times v(W_\tau),$$

s.t.

$$W_1=(W_0-c_1+OP)\times\frac{1}{p(0,1)},$$

and

$$W_\tau=(W_{\tau-1}-c_\tau)\times\frac{1}{p(\tau-1,1)},\ \tau=2,\dots,\omega-x.$$

Threshold set (settling preferred to retaining):

$$\Omega(OP)=\{A_j:\ V_T^{s}(W_0,OP,j)\geq V_T^{r}(W_0,j)\}.$$

Pricing Formula & Generalization

With symmetric information:

$$OP^{\text{sym}}(j) = \sum_{\tau=1}^{\omega-x} \left[\left({}_{\tau-1}p_x{}^{j}(T) - {}_{\tau}p_x{}^{j}(T) \right) \times \frac{F}{(1+R)^{\tau}} - {}_{\tau-1}p_x{}^{j}(T) \times \frac{P}{(1+R)^{\tau-1}} \right]$$

With asymmetric information:

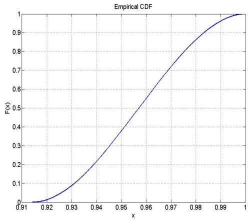
$$\begin{aligned} OP^{\text{e}}(\bar{A}) \quad \triangleq \quad & \arg\max_{z} \left\{ \int_{\Omega(z)} \left(\sum_{\tau=1}^{\omega-x} \left[\left({}_{\tau-1}p_x{}^{j}(T) - {}_{\tau}p_x{}^{j}(T) \right) \times \frac{F}{(1+R)^{\tau}} \right. \right. \right. \\ & \left. \left. \left. - {}_{\tau-1}p_x{}^{j}(T) \times \frac{P}{(1+R)^{\tau-1}} \right] - z \right) f(A_j|\bar{A}) dA_j = 0 \right\} \end{aligned}$$

If allowing settling in future periods:

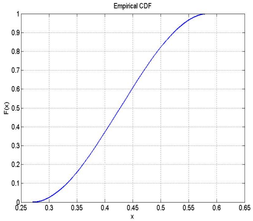
- $V_T^{\text{r}}(W_0, j)$ increases $\Rightarrow$ truncate $\Omega(OP)$ $\Rightarrow$ more significant adverse selection effects
- Systematic mortality risk at population level matters

Life Table Projections

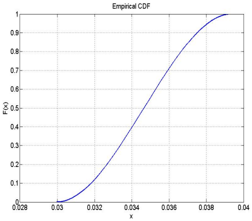
- Population: year 1978 (age 50) $\Leftrightarrow$ year 2008 (age 80)
  - U.S. (female) mortality data from Human Mortality Database
  - Lee-Carter model
  - Year 1978: mortality forecasts for premium setting (data from 1958-1977) $\Rightarrow$ \$16.245 per \$1,000 ($r = 4\%$)
  - Year 2008: mortality forecasts for life settlement pricing (data from 1958-2007)
- Individual: $\gamma = 0.1$, $\frac{A_j+1}{2}$ follows a Beta distribution with parameters $\alpha = \beta = 2$



$_{1}p_{80}$



$_{11}p_{80}$

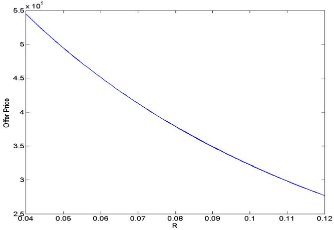


$_{21}p_{80}$

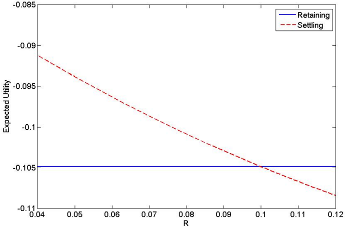
Symmetric Case

- $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\gamma = 1.584$ (cf. Hall and Jones (2007))
- $v(W) = \frac{1+r}{r} \times \frac{(\frac{r}{1+r}W)^{1-\gamma}}{1-\gamma}$
- $W_0 = \$500,000$, $F = \$1,000,000$, $r = 4\%$, $\beta = 1/1.04$

Symmetric Information:



OP^{sym}

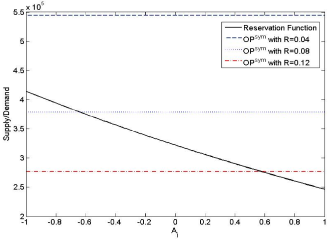


V_T^r and V_T^s

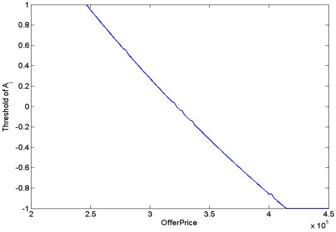
- Settle only when $OP^{sym} \geq \$323,370 \Leftrightarrow R \leq 9.95\%$

Settlement Decision

- By comparing value functions $\Rightarrow$ reservation price for each type A_j
- Calculate OP^{sym} with hurdle rates R at 4%, 8%, and 12%
- When OP^{sym} crosses with the reservation price curve $\Rightarrow$ asymmetric choice from policyholders
- Threshold set: $\Omega(OP) = [A^*(OP), 1]$

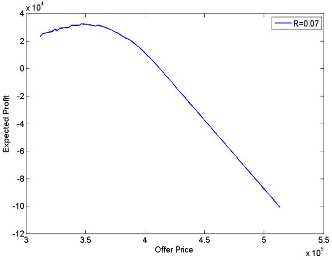


Reservation and Actuarially Fair Prices

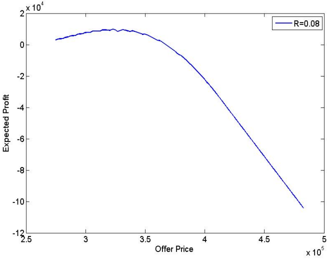


$A^*(OP)$

Equilibrium Offer Prices



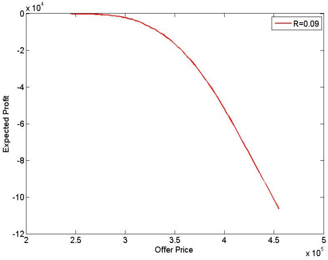
Beta Distribution, $R = 0.07$



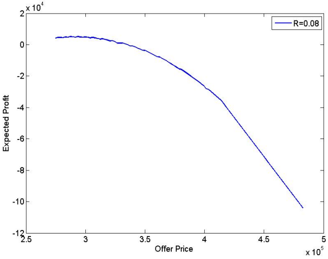
Beta Distribution, $R = 0.08$

- $R = 0.07$: $OP^{e} = OP^{sym} = \$412,680$ (no adverse selection)
- $R = 0.08$: $OP^{e} = \$367,930 < OP^{sym} = \$378,810$ (modest effect of asymmetric information)

Equilibrium Offer Prices



Beta Distribution, $R = 0.09$



Uniform Distribution, $R = 0.08$

- $R = 0.09$: fatal impact from adverse selection $\Rightarrow$ market breaks down
- $R = 0.08$ (uniform A_j): $OP^e = \$339,100$ (stronger adverse selection)

Conclusion

Contributions:

- Effect of asymmetric information on the profit structure of life settlement company

$\Rightarrow$ Applicable pricing formulas for life settlement transactions

$\Rightarrow$ Explanation of the discrepancy between estimated and realized returns

$\Rightarrow$ New angle on the financial analysis of life settlements

$\Rightarrow$ Promote the mortality-linked capital market as a whole

Future projects:

- Calibrate the model parameters
- Sensitivity tests
- Including option to settle later $\Rightarrow$ more severe impact of adverse selection

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Thank you!