Discussion: Pricing Mortality-linked Securities with Dependent Lives under the Multivariate Threshold Life Table

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Agenda

- Summary
- Goal?
- Copula
- Extreme event probabilities
- Other issues

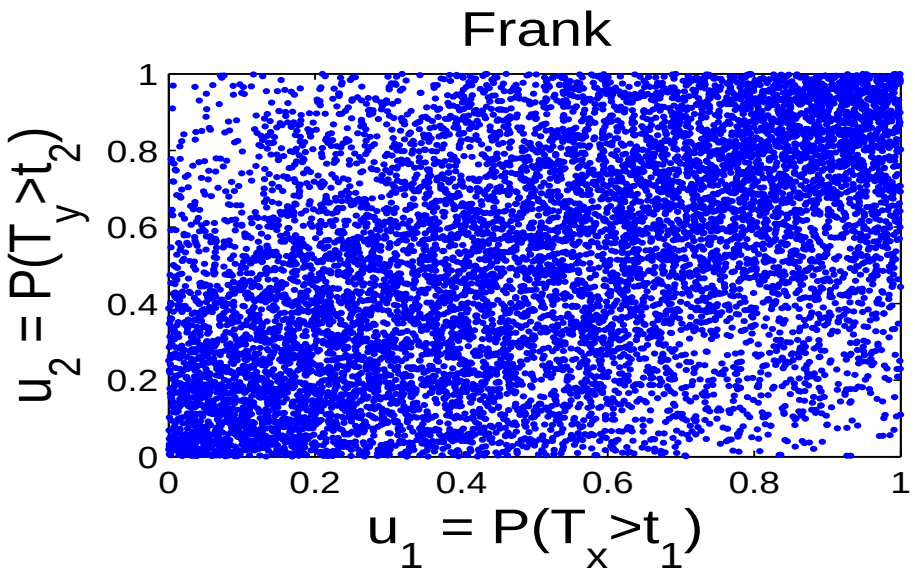
Summary

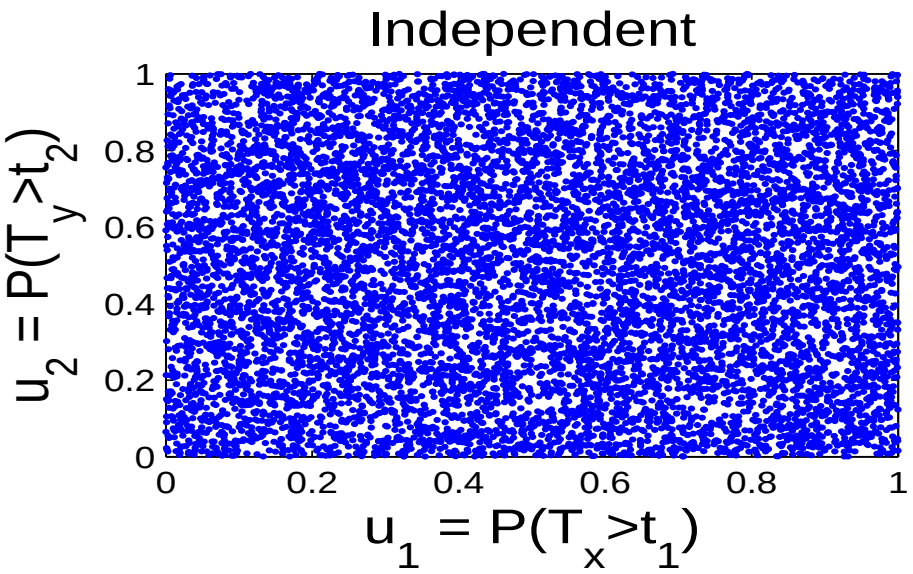
- Pricing annuity products under dependent lives:
  - Marginal distributions:
    i) Gompertz distribution;
    ii) Extreme Value Theory (Generalized Pareto Distribution);
  - Dependence:
    i) Frank copula.

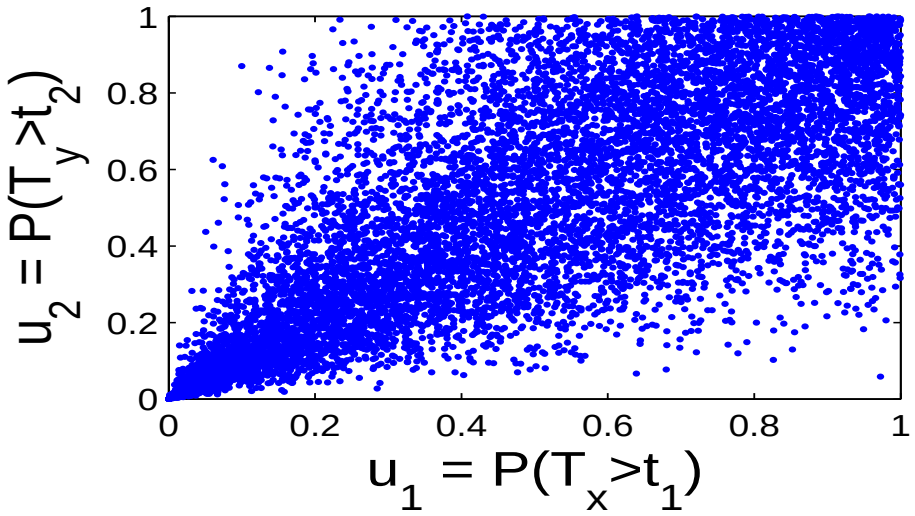
- Results:
  - Annuity values are higher with multivariate threshold life table than without;
  - Dependent mortality reduces joint and survivor annuity values.

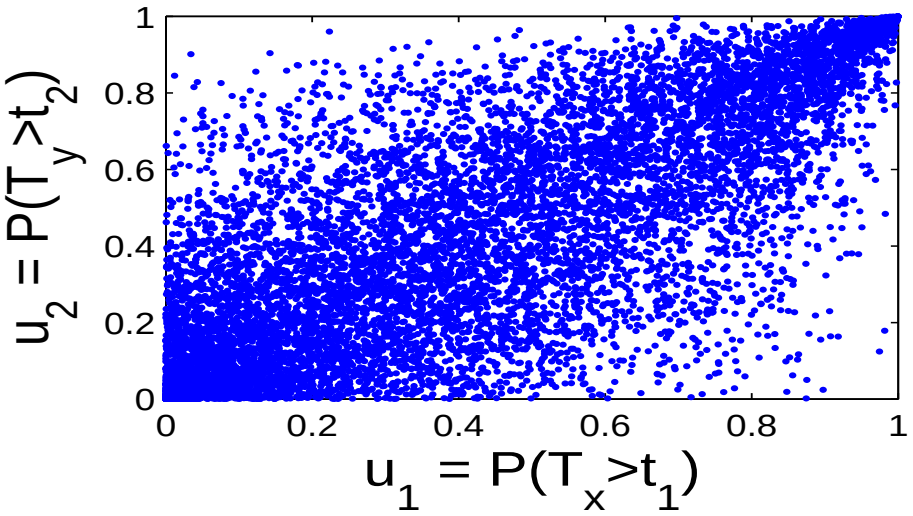
- What is the goal of the paper?
  - Calculating the annuity value of the portfolio of the annuity provider;
  - To set up a general model to calculate annuity values for a (small) group of insureds;
  - Determine the (general) dependence between couple's lifes.
- Can you also use the population life table (see, e.g., Brouhns, 2008)?
- How does the paper fit in the existing literature?

- Minor comment: are mortality rates constant over time? For example: yearly mortality rates may be influenced by hot/cold winter/summer.
- What is the empirical value of Kendall's tau?
- Clayton copula can also cope with negative dependencies;
- Copula choice might be improved;
- How do the properties of the copula fit the data:
  - Tail dependence;
  - Time dependence (see, e.g., Spreeuw, 2006).


Frank
$u_2 = P(T_y > t_2)$
$u_1 = P(T_x > t_1)$
0
0.2
0.4
0.6
0.8
1


Independent
$u_2 = P(T_y > t_2)$
$u_1 = P(T_x > t_1)$
0
0.2
0.4
0.6
0.8
1


Clayton
$u_2 = P(T_y > t_2)$
$u_1 = P(T_x > t_1)$
0
0.2
0.4
0.6
0.8
1


Gumbel
$u_2 = P(T_y > t_2)$
$u_1 = P(T_x > t_1)$
0
0.2
0.4
0.6
0.8
1

Extreme event probabilities

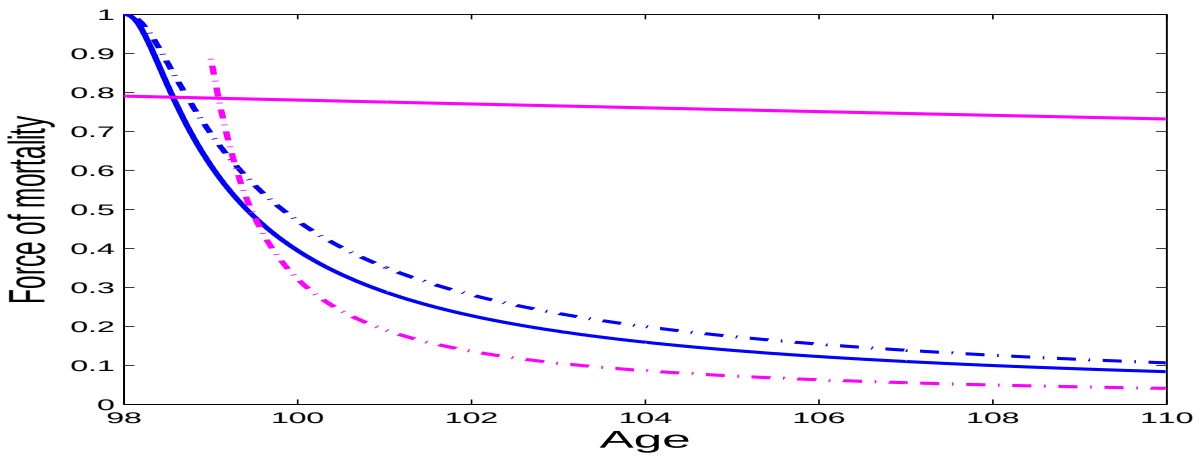
- Generalized Pareto Distribution:

$$F(x) = 1 - \left(1 - \widehat{F}(N)\right) \cdot \left(1 + \epsilon \left(\frac{x - N}{\theta}\right)\right)^{-\frac{1}{\epsilon}}.$$

- How accurate are the estimations?

	N_1	θ_1	ϵ_1	N_2	θ_2	ϵ_2
Bivariate	98	0.11	**0.94**	98	**0.64**	**0.01**
Univariate	98	0.11	**0.73**	99	**0.45**	**2.12**

- How likely are positive values of ϵ, see, e.g., Li, Hardy, and Tan (2009).



Other issues

- Can you perform tests:
  - Extreme value probabilities, i.e., $H_0 : N_1 = N_2 = \infty$ vs $H_1 : N_1 < \infty$, or $N_2 < \infty$;
  - Whether there is dependence;
  - Whether the univariate estimates are different from the bivariate estimates.
- Importance high ages:
  Many $100^+ \rightarrow$ easy to estimate mortality probabilities,
  few $100^+ \rightarrow$ not important for pricing.
- Do same sex annuitants have different dependence?