Real World Pricing of Long Term Contracts

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Abstract. Long dated contingent claims are relevant in insurance, pension fund management and derivative pricing. This paper proposes a paradigm shift in the valuation of long term contracts, away from classical no-arbitrage pricing towards pricing under the real world probability measure. In contrast to risk neutral pricing, the long term excess return of the equity market, known as the equity premium, is taken into account. Further, instead of the savings account, the numéraire portfolio is used, as the fundamental unit of value in the analysis. The numéraire portfolio is the strictly positive, tradable portfolio that when used as benchmark makes all benchmarked nonnegative portfolios supermartingales, which means intuitively that these are downward trending or at least trendless. Furthermore, the benchmarked real world price of a benchmarked claim is defined to be its real world conditional expectation. This yields the minimal possible price for its hedgable part and minimizes the variance of the benchmarked hedge error. The pooled total benchmarked replication error of a large insurance company or bank essentially vanishes due to diversification. Interestingly, in long term liability and asset valuation, real world pricing can lead to significantly lower prices than suggested by classical no-arbitrage arguments. Moreover, since the existence of some equivalent risk neutral probability measure is no longer required, a wider and more realistic modeling framework is available for exploration. Classical actuarial and risk neutral pricing emerge as special cases of real world pricing.

Key words and phrases: long term pricing, real world pricing, risk neutral pricing, numéraire portfolio, law of the minimal price, strong arbitrage, hedge simulation, diversification, liquidity premium.

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