Longevity/Mortality Risk Modeling and Securities Pricing

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July 13, 2010

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Abstract

Securitization of longevity/mortality risk provides insurers and pension funds an effective, low-cost approach to transferring the longevity/mortality risk from their balance sheets to capital markets. The modeling and forecasting of the mortality rate is the key point in pricing mortality-linked securities that facilitates the emergence of liquid markets. The catastrophic longevity jumps and mortality jumps are significant in historical data and have a critical effect on securities pricing. This paper introduces a stochastic diffusion model with a Double Exponential Jump Diffusion (DEJD) process for mortality time-series, which is the first to capture both asymmetric jump features and cohort effect as the underlying reason for the mortality trend. The DEJD model has the advantage of easy calibration and mathematical tractability. The form of the DEJD model is neat, concise and practical. Compared with previous stochastic models with or without jumps, the DEJD model fits the actual data better. To apply the model, the implied risk premium is calculated based on the Swiss Re mortality bond price. The DEJD model is the first to provide a close-form solution to price the q-forward, which is the standard product contingent on the LifeMetrics index for hedging longevity or mortality risk.

Keywords: Longevity Risk; Mortality Risk; Securitization; Double Exponential Jump Diffussion Model; Lee-Carter Framework.

1 Introduction

The terms "longevity risk" and "mortality risk" have attracted the attention of insurance companies, annuity providers, pension funds and investment banks. The definitions are as follows, (Coughlan et al., 2007), longevity risk describes the risk that an individual or group will live longer than expected (their mortality rate will be lower than expected), while mortality risk describes the risk that an individual or group will live shorter than expected (their mortality rate will be higher than expected).

Dramatic improvements in longevity during the 20th century have shown the inadequate management of longevity risk by pension funds. In the last several decades, the life expectancy for populations in the developed world have, on average, been increasing by approximately 1.2 months every year. Globally, life expectancy at birth has increased by 4.5 months per year on average over the second half of the 20th century (Gutterman, England, Parikh and Pokorski, 2002). A recent study of the companies in the UK's FTSE100 index found that underestimation of longevity risk would cause more than double the aggregate deficit from £46 billion to £100 billion (Pension Capital Strategies and Jardine Lloyd Thompson, 2006). In the U.S., the Internal Revenue Service (IRS) has recently established new mortality assumptions for pension contributions, which according to Watson Wyatt, an insurance consulting firm, will increase pension liabilities by 5-10%, (Halonen, 2007). Similarly, Mercer Human Resource Consulting has calculated that the use of up-to-date mortality tables would increase the cost of providing a pension to a male born in 1950 by 8%, Mercer (2006).

On the other side of the risk, many life insurers globally have become concerned about their exposure to catastrophic mortality risk. For example, during the 1918 flu, more than 675,000 excess deaths from the flu occurred between September 1918 and April 1919 in the United States alone, causing a huge jump on the mortality time-series. More recently, H5N1 avian influenza occurred in Hong Kong in 1997, and H1N1 occurred globally in 2009. According to the American Council of Life Insurers, the reserves for U.S. life insurance policies stand at around $1 trillion (ACLI, 2006). There is a possibility that a major pandemic event could trigger insolvency in the life insurance industry, and worsen if catastrophic mortality events coincides with financial downturns.

In the securitization industry, the appearance of Insurance Linked Securities (ILS), facilitates the interaction and combination of the insurance industry and the capital market (Cummins, 2005). Securitization provides the possible approach to offload the non-diversifiable risk from the insurer or pension balance sheet and transfer it to the capital market. This is an efficient, low-cost way to allocate and diversify risk in a much larger pool, constituted by the national or international capital market; it also enhances the risk capacity of the insurance industry, as illustrated by the Catastrophe (CAT) Mortality Bond and other similar derivatives, whose payment depends on the underlying loss indices and the catastrophic mortality event. On the insurance policy holder side, many investment banks have recently been involved in the life-settlement securitization. The investment banks purchased hundreds of thousands of the life insurance policies and repackaged them into bonds, then sold bonds to investors such as pension funds. The payment of the bond depends on the life expectancy of the members in the pool of life insurance policies (Modu, E., 2009).

The pricing of the CAT or life-settlement securities depends on the estimation and forecast of life expectancy or mortality rate, which consider longevity risk and mortality risk. The estimation and forecast of life expectancy or mortality rate also plays the crucial role in longevity/mortality risk management for pension funds or insurers. In this paper, we propose a stochastic model, based on the Brownian Motion process, plus the asymmetric jump process for the estimation and forecast of mortality rate and life expectancy.

Longevity jumps and mortality jumps should be incorporated in the modeling and securitization (Lin, Cox and Pederson 2009), since the jumps are the critical sources of risk which pension funds and insurers should be more cognizant of. The mortality jumps (such as the 1918 flu) have a short-term intensified effect, while the longevity jumps (caused by the pharmaceutical or medical innovation) have long-term gentle effect. The different frequency and intensity of the mortality jumps and the longevity jumps explain the distribution skewness of the mortality time-series increment, which is important but not considered in previous mortality rate models. Considering the asymmetric jump phenomenon of the mortality time-series, our model adopts a compound poisson-Double Exponential Jump Diffusion (DEJD) process to capture the longevity jumps and the mortality jumps, respectively.

Very few studies address the modeling of mortality jumps for securitization. Biffis (2005), Bauer, Borger and Russ (2009) apply affine jump-diffusion process to model force of mortality in a continuous-time framework. Our model incorporates the cohort effect, which captures the the mortality time-series and adjusts it to fit different age groups. The model with cohort effect captures the feature of the historical mortality time-series more accurately. Chen, Cox and Pederson (2009) model the jump process with a compound Poisson normal jump diffusion process. Our model makes the contribution of applying the double exponential jump diffusion which differentiates the longevity jumps and the mortality jumps. This captures the distribution skewness of the mortality time-series increment and offers better fitness. Lin, Cox and Pederson (2009) propose a model to accommodate both longevity jumps and mortality jumps. While our model has fewer parameters, more concise specification, and can be easily parameterized and applied to the securitization.

Our model incorporates the advantages of the Lee-Carter framework, which describes the mortality time-series and considers the cohort effect, making the age-specific adjustment for different age groups. The adjustment is critical for the model, since the mortality improvement and extreme positive or negative event (such as influenza pandemic) has different intensity level on different age groups. In this way, the Lee-Carter model appropriately describes the three dimensional surface of the mortality rate with respect to time horizon and age group horizon. The Lee-Carter framework has been extended by Brouhns, Denuit and Vermunt (2002), Renshaw and Haberman (2003), Denuit, Devolder and Goderniaux (2007), Li and Chan (2007), Chen and Cox (2009).

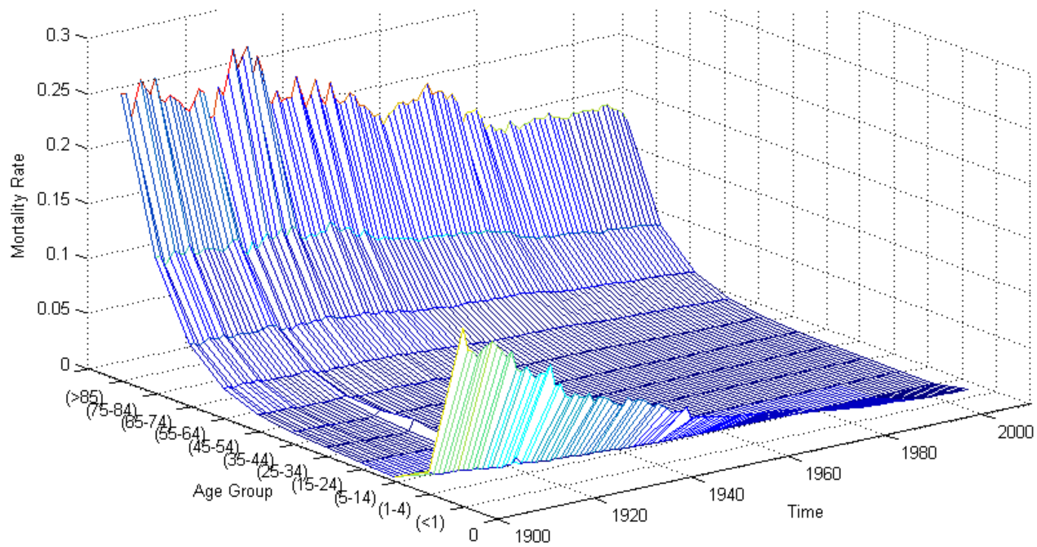
We test our model with historical data and make model fitness comparisons with previous models. The results clearly illustrate the advantage in fitness. In application, we use the first CAT mortality bond, the Swiss Re Catastrophe Mortality Bond (2003) to calculate the implied risk premium, with two types of changing measure approach. We implement our model to price the q-forward as an example, which illustrates the benefit of our model for providing a closed-form pricing solution for standard structure mortality linked securities. q-forward is proposed by JP Morgan based on the LifeMetrics index, which has the potential to be future standardized contract and helps to create a liquid market. Beyond q-forward, our model can provide closed-form pricing solution to all the mortality-linked securities with cash flows that are linear functions of the mortality rate for each period.

Our paper is organized as follows. In Section 2 we introduce the historical mortality rate and describe the mortality rate age-specific features and the mortality time-series. In Section 3, we describe the model requirement and specification which includes a baseline Brownian Motion process and a coumpound poisson Double Exponential Jump Diffusion process. In Section 4, The numerical method is applied to calibrate the model and compare the fitness of the model with previous ones. In Section 5, we use the Swiss Re Catastrophe Mortality Bond as the benchmark product to calculate the implied market price of risk. Based on the implied market price of risk and our model, we price the standard market product q-forward as a pricing example. We conclude and discuss the paper in Section 6.

2 Data description

The historical data come from HIST290 National Center for Health Statistics, the same data used in Chen and Cox (2009). We use the same data in order to facilitate the model fitness comparison in Section 4. The data lists death rates per 100,000 population for selected causes of death. Death rates are tabulated for age group (<1), (1-4), (5-14), (15-24), then every 10 years, to (75-84), and (>85), including both sex and race categories. Selected causes for death include major conditions such as heart disease, cancer, and stroke. Figure 1 shows the mortality rate for different age groups and years. Figure 2 shows the comparison of the mortality rate for several sample age groups, including relatively older groups and younger groups.

Figure 1. 1900-2004 Mortality Rate

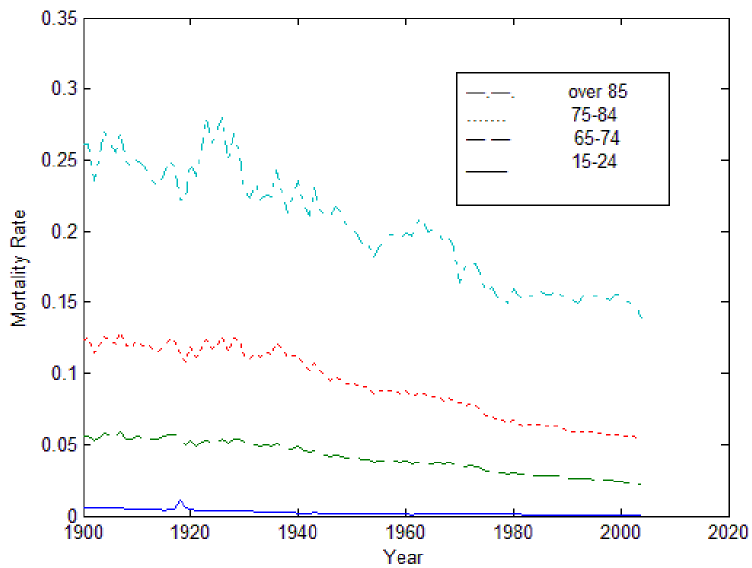


We can observe clearly the two properties of the mortality rate trend from Figure 1 and Figure 2.

In Figure 1, the downward trend indicates that the mortality rate follows a decreasing trend during 1900-2004. For example, in the over 85 old age group, the mortality rate decreases from 0.26 to 0.14, while in the young age group such as 15-24, the mortality rate decreases from 0.006 to 0.0008. The decreasing trend shows the improvement of the life length, or longevity in all age groups.

In Figure 2, the change in the mortality rate in the older-age groups is more significant with a steeper downward trend than that in the younger-age groups. For example, the mortality rate decreases 0.12 in the older age group, over 85. During the same time period, the mortality rate decreases only 0.0052 in the younger-age group, 15-24. The comparison of the steepness of the mortality surface shows that the improvement of the longevity of the older-aged population is more significant than that of the younger-aged population. We can observe the variability and dynamics on the mortality trend, so the model is required to capture the trend dynamics.

Figure 2. Comparison of the Age Group Mortality Rates



3 Model

3.1 Model Framework and Requirement

The basic requirement of the mortality model is to capture the features described above. Various mortality rate models have been provided by previous research. The majority of models are based on the Lee-Carter one-factor model (Lee and Carter, 1992). These models extend the Lee-Carter model to a two-factor model (Blake, Carins, and Dowd, 2006), or engage stochastic process in the factor (Dahl and Moller, 2004), or encompass the jump with a function of the stochastic process to characterize the extreme outliers in the mortality time series (Cox, Lin, and Wang 2006; Chen and Cox 2008).

In the Lee-Carter framework, the mortality rate $\mu_{x,t}$ on different age x and time t is decomposed into age-specific parameters a_x, b_x and mortality time-series k_t.

$$\ln(\mu_{x,t}) = a_x + b_x k_t + e_{x,t} \tag{1}$$

$$\mu_{x,t} = \exp(a_x + b_x k_t + e_{x,t}) \tag{2}$$

Where a_x represents the age groups shift effect, and e^{a_x} is the general shape across the age of the mortality schedule. b_x represents the age group's reaction effect to mortality time-series k_t. In other words, the b_x profile tells us which group of mortality rates decline rapidly and which group of mortality rates decline slowly in response to changes in k_t. And $e_{x,t}$ captures the age group's residual effect not reflected in the model. Lee-Carter model suggests a two-stage procedure Single Value Decomposition (SVD) to use historic data of $\mu_{x,t}$ to estimate the age-specific parameters a_x, b_x, and generate the mortality time-series k_t .

To implement the SVD procedure, first, we need to normalize the condition that set k_t sums to 0 and b_x sums to 1. Then a_x must equal the average over time of $\ln(\mu_{x,t})$.

$$a_x = \frac{1}{T}\sum_{t=1}^{T} \ln(\mu_{x,t}) \tag{3}$$

Furthermore, k_t must (or almost) equal the sum over age of $(\ln(\mu_{x,t}) - a_x)$, since the sum of the b_x has been chosen to unity. This is not an exact relation, however, since the error terms will not in general sum to 0 for a given age. Then, each b_x can be found by regressing, without a constant term, $(\ln(\mu_{x,t}) - a_x)$ on k_t separately for each age group x.

In the second stage, re-estimate mortality time-series k_t iteratively, given the estimation of a_x and b_x in the first step, enables the actual sum of death at time t (left-hand side) to equal the implied sum of deaths at time t (right-hand side).

$$D_t = \sum_x (P_{x,t} \exp(a_x + b_x k_t)) \tag{4}$$

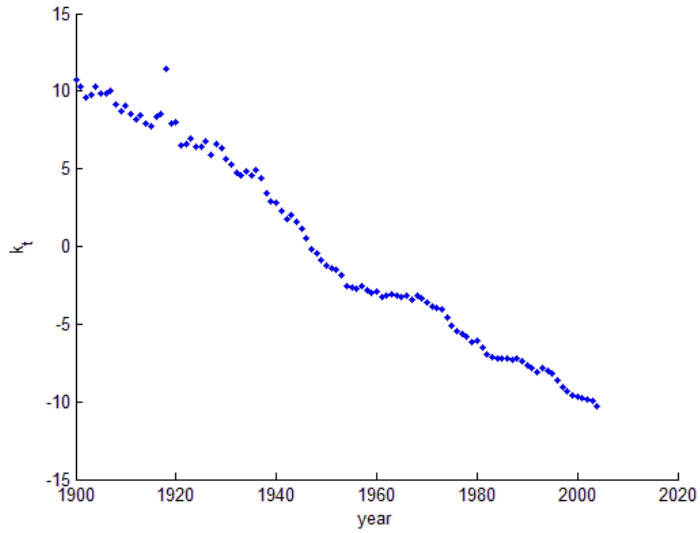
where D_t is the actual sum of deaths at time t , and $P_{x,t}$ is the population in age group x at time t.

Implementing the SVD two-stage procedure with data from historic U.S. mortality rates over 1900-2004, we get the fitted a_x, b_x, in Table 1, and the mortality time-series k_t in Figure 3. The decreasing trend of mortality time-series k_t shows the improvement of mortality along the time as described. Figure 3 also shows the big jump in 1918 which is caused by flu and other asymmetric jumps around 1920, 1943, etc.

Table 1. Fitted Value for Age-Specific Parameters a_x and b_x during 1900-2004

Age Group	a_x	b_x
<1	-3.4087	0.1455
1-4	-6.2254	0.1960
5-14	-7.1976	0.1492
15-24	-6.2957	0.0994
25-34	-5.9923	0.1044
35-44	-5.4819	0.0855
45-54	-4.7799	0.0608
55-64	-4.0137	0.0468
65-74	-3.2347	0.0426
75-84	-2.4196	0.0409
>85	-1.6119	0.0290

Figure 3. The Mortality Time-Series k_t

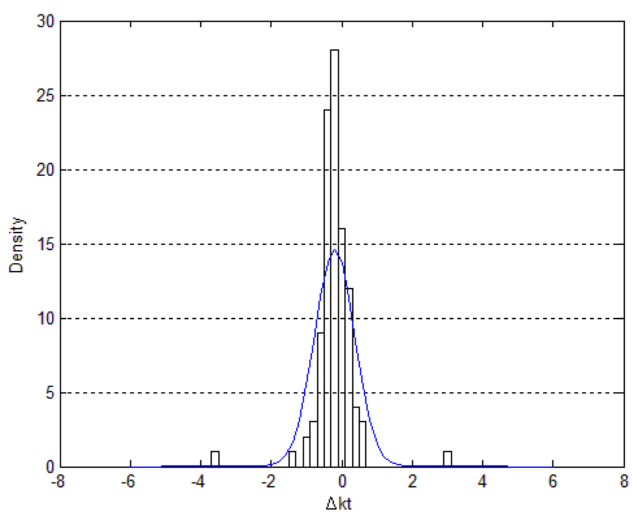


Now, we need a model to capture the features of the shape, the trend and the jumps of the mortality time-series k_t. First, the model should engage the stochastic process to describe the k_t time series, which has proved to be better than the model without stochastic process, in (Dahl and Muller, 2004). Second, shown in Figure 3, since k_t includes both positive and negative values, geometric Brownian motion, which will not generate a negative value from the positive starting value, does not fit the process. The specification of the Brownian motion can be selected to fit the k_t time series. Third, we can observe from Figure 3 that the jumps are transient, not permanent. For example, the sudden increase of mortality rate in 1918, caused by the flu, falls back to the normal condition in the second year.

Beyond the three points on the model specification listed above, the asymmetric jump phenomenon needs to be considered. As seen in Figure 3, the positive jumps (the suddenly remarkable increase in the mortality time-series) are of high-severity and low-frequency, while the negative jumps (the suddenly remarkable decrease in the mortality time-series) are of low-severity and high-frequency. Hence, the model that involves a jump process with a symmetric normal distribution (Chen and Cox, 2009) does not capture the asymmetric jump phenomenon. From the biological and demographic perspectives, the positive jumps (mortality jumps) can be explained by sudden catastrophic reasons (e.g., earthquake, hurricane) or critical diseases, such as the extreme positive jump caused by flu in 1918. The negative jumps (longevity jumps) are associated with multiple biological and health improvement causes. The improvement of mortality due to health or biological reasons is moderate and does not show dramatic effect. As a result, the jumps are not symmetric and the severity cannot be characterized by a normal distribution.

The descriptive statistics of $\Delta k_t = k_{t+1} - k_t$ shows asymmetric leptokurtic features. The skewness of Δk_t equals to -0.451. In other words, Δk_t distribution is skewed to the left, and has a higher peak and two heavier tails than those of a normal distribution, which we can also observe in Figure 4.

Figure 4. Comparison of Actual Δk_t Distribution and Normal Distribution



In Figure 4, the histogram represents the distribution of actual Δk_t, and, as can be seen, Δk_t cannot be fitted by normal distribution. Hence, the Brownian motion process cannot be used to describe mortality time-series k_t.

To incorporate the leptokurtic feature of the Δk_t distribution, the analysis here incorporates a Double Exponential Jump Diffusion (DEJD) model to capture both the positive jumps and negative jumps of the k_t process. Compared to Cox, Lin, Perdersen(2008), which also captures the positive jumps and negative jumps separately, our model has a concise specification and an easy approach for calibration. What is more, unlike Cox et. al. (2008), our model has a closed-form solution for the forecast of the future mortality rate, which facilitates mortality securities pricing.

3.2 The Model Specification

To capture the features of the mortality time-series k_t, and to account for the tractability and the calibration of the model, we set the model specification to describe Δk_t in the approximate continous-time model of dk_t as the followings.

The dynamics of the mortality time-series k_t is specified as follows:

$$dk_t = \alpha dt + \sigma dW_t + d(\sum_{i=1}^{N(t)}(V_i - 1)) \tag{5}$$

where W_t is a standard Brownian motion, $N(t)$ is a Poisson process with rate λ, and λ describes the frequency of the jumps. The larger the λ, the more times that jumps occur in the mortality time-series. And V_i is a sequence of independent identically distributed $(i.i.d)$ nonnegative random variables, $s.t.$ $Y = \log(V)$ has an asymmetric double exponential distribution with the density

,

$$\begin{aligned} &f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geqslant 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y<0\}}, \\ &\eta_1, \eta_2 > 0, \quad p, q \geqslant 0, \qquad p + q = 1. \end{aligned} \tag{6}$$

The p, q represent, respectively, the proportion of positive jumps (the suddenly remarkable increase in the mortality time-series) and negative jumps (the suddenly unremarkable decrease in the mortality time-series) among all jumps. So, $p\lambda$ is the frequency of positive jumps and $q\lambda$ is the frequency of negative jumps. η_1 and η_2 describe the positive jumps severity and the negative jumps severity separately. The larger the η_1, the smaller the positive jumps severity, and the same feature for negative too. In this way, the positive jumps and negative jumps are captured by similar distributions with different parameters, which match the asymmetric mortality time-series k_t and the leptokurtic feature of dk_t

The model specification with the double-exponential distribution has the advantage of mathematical tractability from which the closed-form formula of expected future mortality rate can be derived. Also, the double-exponential distribution is widely implemented in the stock price jump-diffusion process, on which the closed-form solution for options and other securities can be generated (Kou, 2004). In this paper, the closed-form solution of expected future mortality rate is presented as an example in (12).

Considering the risk neutral measure, then (5) becomes

$$dk_t = (\alpha^* - \lambda^*\gamma^*)dt + \sigma dW^* + d(\sum_{i=1}^{N^*(t)} (V_i^* - 1)) \tag{7}$$

$$\gamma^* = E^*[V^*] - 1 = \frac{p^*\eta_1^*}{\eta_1^* - 1} + \frac{q^*\eta_2^*}{\eta_2^* + 1} - 1 \tag{8}$$

Integrate (7)

$$k_s = k_0 + (\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^*\gamma^*)s + \sigma W_s^* + \sum_{i=1}^{N^*(s)} Y_i^* \tag{9}$$

According to the characteristic function

$$E^*[e^{\theta(k(s)-k_0)}] = \exp[G(\theta)s] \quad \text{or}$$

$$E^*[e^{\theta k(s)}] = \exp(\theta k_0)\exp[G(\theta)s] \tag{10}$$

where

$$G(\theta) = \theta(\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^*\gamma^*) + \frac{1}{2}\theta^2\sigma^{*2} + \lambda^*(\frac{p^*\eta_1^*}{\eta_1^* - \theta} + \frac{q^*\eta_2^*}{\eta_2^* + \theta} - 1) \tag{11}$$

Using b_x for θ in (10), the closed-form expression for the expected future mortality rate $\mu_{x,t}$ is derived as

$$\begin{aligned} E^*[\mu_{x,t}] &= \exp(a_x) \times E^*[\exp(b_x k_t)] \\ &= \exp(a_x + b_x k_0 + b_x t(\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^*\gamma^*) + \frac{1}{2}b_x^2\sigma^{*2}t + \lambda^* t(\frac{p^*\eta_1^*}{\eta_1^* - b_x} + \frac{q^*\eta_2^*}{\eta_2^* + b_x} - 1)) \end{aligned} \quad (12)$$

The formula (12) can be used to calculate the expected future mortality rate directly with parameters $\{\lambda^*, p^*; \eta_1^*, \eta_2^*; \alpha^*, \sigma^*\}$, which is much faster and more convenient than using simulation to project and average the paths of future mortality rates. This model is especially suitable for pricing the mortality linked securities whose cash flow each period is a linear function of the mortality rate (e.g., the q-forward). We will discuss the example of q-forward pricing in section 4.4.

4 Numerical Solution

4.1 Parameter Calibration

The disentangling of jumps from diffusion is a serious difficulty in calibration of the underlying variable (like stock price) evolution process. The increments of underlying variable evolution process are supposed to be captured by the diffusion process, with a few extreme increments captured by jumps. However, the addition of the jump process may yield the wrong calibrated parameters which leads to high frequency of jumps and small severity of jumps. The double-exponential jump diffusion model for dk_t faces the same problem. A calibration method is needed to generate right parameters of low frequency and large severity for jumps. Ait-Sahalia and Hansen (2004) demonstrate that Maximum Likelihood Estimation (MLE) has advantages in disentangling jumps from diffusion. Meanwhile, the double-exponential jump diffusion is a linear process with independent increments and an explicit transition density, which fortunately satisfies the requirement of a complete specification of the transition density for using MLE. Therefore, we choose the MLE method to calibrate parameters $\{\lambda, p; \eta_1, \eta_2; \alpha, \sigma\}$ with dk_t time series, beyond the methods of GMM, the simulated moment estimation, and MCMC.

Let C=$\{k_0, k_1, ..., k_T\}$ denote the mortality time-series, at equally spaced times $t = 1, 2, ..., T$.

The one period increment $r_i = \Delta k_i = k_i - k_{i-1}$ is *i.i.d.* The unconditional density of one period increment $f(r)$ is:

$$f(r) = e^{-(\lambda_u+\lambda_d)} f_{0,0}(r) + e^{-\lambda_u} \sum_{n=1}^{\infty} p(n,\lambda_d) f_{0,n}(r) \tag{13}$$

$$+e^{-\lambda_d} \sum_{m=1}^{\infty} p(m,\lambda_u) f_{m,0}(r) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} p(n,\lambda_d) p(m,\lambda_u) f_{m,n}(r) \tag{14}$$

where $p(n,\lambda_d) = e^{-\lambda_d}\lambda_d/n!$, $p(m,\lambda_u) = e^{-\lambda_u}\lambda_u/m!$ and $f_{m,n}(r)$ is the conditional density for one period increment, conditional on the given numbers of up and down jumps (m,n).

$$f_{0,0}(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(r-\mu+\frac{1}{2}\sigma^2)^2} \tag{15}$$

$$f_{0,n}(r) = \frac{\eta_d^n}{(n-1)!\sqrt{2\pi}\sigma} \int_{-\infty}^{0} (-x)^{(n-1)} e^{(\eta_d x - \frac{1}{2\sigma^2}(r-x-\mu+\frac{1}{2}\sigma^2)^2)} dx \tag{16}$$

$$f_{m,0}(r) = \frac{\eta_u^m}{(m-1)!\sqrt{2\pi}\sigma} \int_{0}^{\infty} (x)^{(m-1)} e^{(-\eta_u x - \frac{1}{2\sigma^2}(r-x-\mu+\frac{1}{2}\sigma^2)^2)} dx \tag{17}$$

$$f_{m,n}(r) = \frac{\eta_u^m \eta_d^n}{(m-1)!(n-1)!\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{0\wedge t} (-x)^{n-1} (t-x)^{m-1} e^{(\eta_u+\eta_d)x} dx \right) \tag{18}$$

$$\times e^{-\eta_u t} e^{-\frac{1}{2\sigma^2}(r-x-\mu+\frac{1}{2}\sigma^2)^2} dt \tag{19}$$

The log-likelihood given T equally spaced increment observations is:

$$L(C;\lambda_u,\lambda_d;\eta_1,\eta_2;\alpha,\sigma) = \sum_{i=1}^{T} \ln(f(r_i))$$

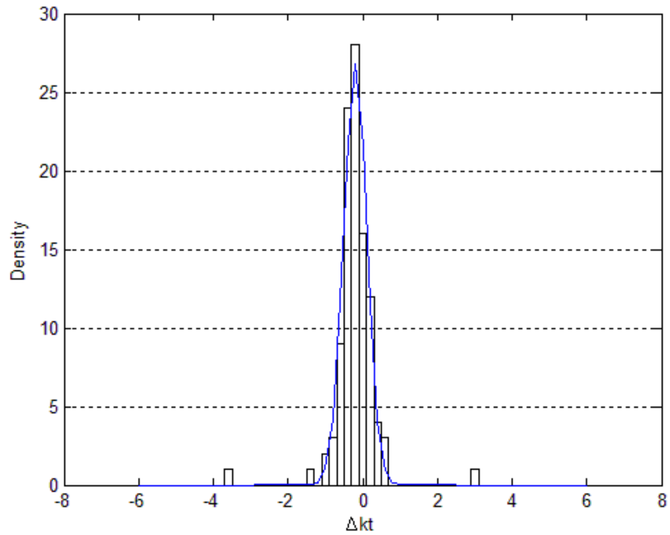
where $\lambda = \lambda_u + \lambda_d$, and $p = \frac{\lambda_u}{\lambda}$, then we can calibrate parameters $\{\lambda, p;\ \eta_1, \eta_2;\ \alpha, \sigma\}$.

After computation, we get $\{\lambda_u, \lambda_d;\ \eta_1, \eta_2;\ \alpha, \sigma\}$={0.029,0.035,0.71,0.75;-0.20,0.31}, and $\{\lambda, p;$ $\eta_1, \eta_2;\ \alpha, \sigma\}$={0.064,0.45;0.71,0.75;-0.20,0.31} and maximum likelihood value $L = -49.95$. η_1 and η_2 describes the severity of positive jumps (mortality jump) and negative jumps (longevity jump), respectively. The larger η represents the smaller severity. $\eta_1 = 0.71 < 0.75 = \eta_2$ verifies that the severity of positive jumps is larger than the severity of negative jumps, as observed in Figure 3. λ_u and λ_d describes the frequency of positive jumps (mortality jump) and negative jumps (longevity jump), respectively. The larger λ represents the larger frequency. $\lambda_u = 0.029 < 0.035 = \lambda_d$ verifies that the frequency of positive jumps is smaller than the frequency of negative jumps, as observed in Figure 3.

4.2 Model Comparison

Figure 5 shows how the DEJD model fits the actual increment of mortality rate Δk_t, by comparing the distribution generated by the DEJD model calibrated by historical data and the actual distribution of Δk_t. Comparing Figure 5 with Figure 4, The DEJD model approximates the distribution of increment of mortality rate Δk_t much better than the commonly used Lee-Carter Brownian Motion model. The mean of the distribution of DEJD and the Brownian Motion is the same, ($\mu_{DEJD} = \mu_{BM} = -0.20$), while the standard deviation of the distribution of DEJD ($\sigma_{DEJD} = 0.31 < \sigma_{BM} = 0.57$) is significant less than the Brownian Motion. It is exactly the reason that the DEJD model is more appropriate to fit the actual distribution, which can be directly observed from the comparison of the two figures. The underlying reason is that the Lee-Carter model includes the outliers in the Brownian Motion diffusion process, which causes the calibrated σ_{BM} to be larger. And the Brownian Motion diffusion is appropriate to capture the normal distribution shape without fat tail and high peak, which is not this case. In our DEJD model, we include the outliers in the asymmetric jump diffusion part, which enables the calibrated σ_{DEJD} smaller than σ_{BM} and improves the fitness.

Figure 5. Comparison of Actual Δk_t Distribution and DEJD Distribution



Next, the DEJD model is compared with both Lee-Carter Brownian motion model and the normal jump diffusion model (Chen and Cox, 2009). For model selection, we adopt the widely used Bayesian Information Criterion (BIC) proposed by Schwarz (1978). Unlike the significance test, BIC allows comparison of more than two models at the same time and does not require that the alternatives to be nested. BIC is a "conservative" criterion since it heavily penalizes over parameterization (Ramezani and Zeng, 2007).

Suppose the kth model M_k, has parameter vector θ_k, where θ_k consists of n_k independent parameters to be estimated. Denote θ'_k as the MLE of θ_k. Then, BIC for Model M_k is defined as:

$$BIC_k = -2\ln f(C|\theta'_k, M_k) + n_k \ln(m), \tag{20}$$

where m is the number of observations in data set C and $f(C|\theta'_k, M_k)$ is the maximized likelihood function. Clearly the best "fit" model is one with the smallest BIC. Given the maximum likelihood function value in (Chen and Cox, 2008),

Table 2. Comparison of Model Fitness

	Number of Parameters	ln(likelihood)	BIC
Double Exponential Jump Diffusion Model (DEJD Model)	6	-49.95	127.76
Normal Jump Diffusion Model (Chen-Cox Model)	5	-62.52	148.26
No Jump Diffusion Model (Lee-Carter Model)	2	-94.27	197.83

The underlying reasons that our DEJD model fits the data better are as followings.

1. The outliers on the mortality time-series cause the fat tail and the high peak in the increment Δk_t distribution which is not appropriate to be describe with normal distribution. Lee-Carter model treats the outliers the same way as other points on the mortality time-series evolution process. As a result, the outliers enhance the variability of the process and cause the overestimate of standard deviation σ. Our DEJD model apply compound poisson Double Exponential Jump Diffusion process to separate the Brownian Motion diffusion process and asymmetric jump diffusion process, which solves the problem of mismatching the fat tail and high peak with the normal distribution and hence provides better fitness.

2. The Chen-Cox model applies the normal jump diffusion model which is composed of the Brownian Motion diffusion process and the normal jump diffusion process. The model treats the outlier with the normal distribution. Actually, the positive outlier (or jump) and the negative outlier (or jump) are caused by different biological and technical reasons. The positive outliers (caused by the pandemic influenza, like 1918 flu) have a short-term intensified effect, while the negative outliers (caused by the pharmaceutical or medical innovation) have long-term gentle effect. The different frequency and intensity of the positive outliers and the negative outliers explain the distribution skewness of the mortality time-series increment, which is not appropriate to be described with the symmetric normal jump diffusion. Our model employs an asymmetric jump diffusion to separately describe the features of the positive outliers and the negative outliers, which solves the problem of skewness of the increment distribution and hence provides better fitness.

4.3 Implied Market Price of Risk

As the other insurance product, such as the annuity, the longevity risk contingent securities are priced in an incomplete market. Hence, the risk premium should be considered in the pricing issues, which represents the price that pension funds or insurers are willing to pay to transfer the longevity or mortality risk. In the previous research papers, (Blake, Cairns, Dowd, MacMinn 2006), (Chen, Cox 2009), the Swiss Re Mortality Catastrophe Bond is used to calculate the implied mortality risk premium, given its payment structure and issue price.

The Swiss Re Mortality Catastrophe Bond

The Swiss Reinsurance company issued the first mortality risk contingent securitization in December 2003. When the bond is triggered by a catastrophic evolution of death rates of a certain population, the investors incur the loss in principal and interest. The bond provides the investor higher yield as a compensation for the mortality risk they take. The bond was issued through a special purpose vehicle (SPV) called Vita Capital, which enabled Swiss Re to remove extreme catastrophic risk from its balance sheet.

The bond had a maturity of three years, a principal of $400m, the coupon rate of 135 basis points plus the LIBOR. The mortality index, M_t, was a weighted average of mortality rates over five countries, males and females, and a range of ages. The principal was repayable in full only if the mortality index did not exceed 1.3 times the 2002 base level during any year of the bond's life, and was otherwise dependent on the realized values of the mortality index. The precise payment schedules were given by the following $f_t(\cdot)$ functions:

$$f_t(\cdot) = \left\{ \begin{array}{ll} \text{LIBOR + spread} & t = 1, ..., T-1 \\ \text{LIBOR + spread + } \max\{0, 100\% - \sum_t L_t\} & t = T \end{array} \right\},$$

$$L_t = \begin{cases} 0\% \\ [(M_t - 1.3M_0)/(0.2M_0)] \times 100\% \\ 100\% \end{cases} \text{if} \begin{cases} M_t < 1.3M_0 \\ 1.3M_0 \leq M_t \leq 1.5M_0 \\ 1.5M_0 < M_t \end{cases} \text{for all } t \qquad (21)$$

4.3.1 Risk-Neutral Pricing

Risk-neutral pricing method is used by Milevsky and Promislow (2001) and by Cairns, Blake, and Dowd (2006a). The method is derived from the financial economic theory that posits it even in an incomplete market. If the overall market is no arbitrage, there exists at least one risk-neutral measure Q for calculating fair prices. We apply the approach in Cairns, Blake, and Dowd (2006b), which assumes the market price of mortality risk is constant and estimates it from the EIB/BNP LB in November, 2004. The more sophisticated assumptions about the dynamics of the market price is pointless, since the available issued mortality linked securities are rare and the data are limited. However, as the mortality linked securities liquid market develops, the more accurate market price of risk can be calculated based on the adequacy of deals and data.

We can derive the implied market price of risk ζ based on the actively traded mortality linked securities on the market whose fair price is already known, and then apply the same ζ to price the unknown mortality linked securities. In the previous research, the annuity price (Cox and Lin, 2006) or the mortality bond price (Chen and Cox, 2009), is applied as the known traded price. In this paper, we use Swiss Re mortality catastrophe bond as the known price to calculate ζ, then to price the q-forward with ζ as an implementation example of our DEJD model. The set of implied market prices of risk $\zeta = \{\zeta_1, \zeta_2, \zeta_3\}$ is in correspondence to the Brownian motion, the positive jump severity and the negative jump severity, $\{\alpha^*, \eta_1^*, \eta_2^*\}$. Since the mortality linked securities are priced in the existing incomplete market, the value of ζ or the risk-neutral measure Q is not unique. We have only one mortality linked security but need to calculate three market prices of risk. Following (Cairns, Blake, and Dowd, 2006b), we can estimate the set of ζ by changing one and fixing the rest.

Assume the market prices of risk set $\zeta = \{\zeta_1, \zeta_2, \zeta_3\}$. $\alpha^* = \alpha + \zeta_1$; $\eta_1^* = \eta_1 + \zeta_2$; $\eta_2^* = \eta_2 + \zeta_3$. We can estimate component ζ by changing only one and fixing the rest.

The algorithm below is similar to the traditional procedure for calculating the market price of risk with Wang Transform approach (Chen and Cox, 2009):

Step 1. Based on the known 2003 mortality time-series, simulate 10,000 times the future mortality time-series $k(t)$ for 2004-2006, using the DEJD model (5) with the calibrated parameter set

$\{\lambda, p;\ \eta_1, \eta_2;\ \alpha, \sigma\}$={0.064,0.45;0.71,0.75,-0.20,0.31}, and the initial assumed set $\zeta = \{\zeta_1, \zeta_2, \zeta_3\} = \{0, 0, 0\}$, with the risk-neutral transform function.

Step 2. Calculate the mortality rate $\mu_{x,t}$ by the formula (2) and calculate the average μ_t based on year 2000 standard population and corresponding weights.[1]

Step 3. Calculate expected value of the principal payment in every period T by the formula, $E_T^*[payment] = \$400,000,000 \times [\max(1 - \sum_{t=2004}^{2006} L_t, 0)]$, L_t follows (21). The coupon payment in every period is calculated based on the par spread plus 1.35% risk premium.

Step 4. Iteratively adjust the market price of risk set ζ, and repeat step.1-step.3 until the discounted expected value of the coupon payment in 2004-2006 plus the principal payment in 2006 equal the face value of the mortality bond $400,000,000.

Table 4. Implied Market Prices of Risk by Risk-Neutral Approach

ζ_1	ζ_2	ζ_3
6.15	0	0
0	0.21	0
0	0.18	0.18

4.4 Example: q-forward pricing

q-forwards are proposed by JP-Morgan as simple capital markets instruments for transferring longevity risk and mortality risk. q-forwards enable pension funds to hedge against increasing life expectancy of plan members and life insurers to protect themselves against significant increases in the mortality of policyholders. Similar to other forwards, q-forwards are securities involving

[1] The year 2000 standard population and corresponding weights is based on the technique notes of the NCHS report GMWK293R. The weight is 0.013818 for (<1) age group, 0.055317 for (1–4) age group, 0.145565 for (5–14) age group, 0.138646 for (15–24) age group, 0.135573 for (25–34) age group, 0.162613 for (35–44) age group, 0.134834 for (45–54) age group, 0.087247 for (55–64) age group, 0.066037 for (65–74) age group, 0.044842 for (75–84) age group, and 0.015508 for (>85) age group.

the exchange of the realized mortality of a population at some future date, in return for a fixed mortality rate agreed at inception. q-forwards form the basic building blocks from which many other complex securities can be constructed. A q-forward provides a type of standardized contract which could help to create a liquid market. A set of q-forwards that settle based on the LifeMetrics Index could fulfill this role. Since the investors require a risk premium to take on longevity risk, the mortality forward rates at which q-forwards transact will be below the expected, or "best estimate" mortality rates.

Figure 6. Mortality Risk Hedge

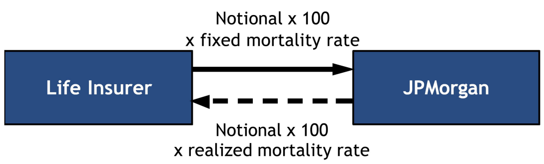
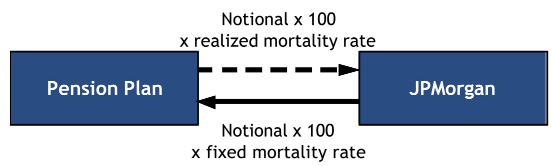


Figure 7. Longevity Risk Hedge



Source: JP Morgan, q-Forward

A q-forward contract to hedge the mortality risk of a life insurer is that a life insurer pays fixed mortality rate to JP Morgan and JP Morgan pays realized mortality rate to the life insurer. The life insurer receives the payment $Notional\ Amount * (q_{\text{realized}} - q_{\text{fixed}}) * 100$, as Table 5 shows as an example of the q-forward structure. When the mortality rate increase, the JP Morgan pays more (realized rate minus fixed rate) to the life insurer which covers the loss incurred by life insurer due to the mortality risk. A q-forward contract to hedge the mortality risk of a pension fund is that a pension fund pays realized mortality rate to JP Morgan and JP Morgan pays the fixed rate to the pension fund. The pension fund receives the payment $Notional\ Amount * (q_{\text{fixed}} - q_{\text{realized}}) * 100$. When the mortality decrease, the JP Morgan pays more (fixed rate minus realized rate) to the pension fund which covers the loss incurred by the pension fund due to the longevity risk. In this way, pension funds who long the longevity risk can transfer the risk to the investors who want to short the longevity risk. And life insurers who long the mortality risk can transfer the risk to the investors who want to short the mortality risk.

Table 5. Example of q-Forward Structure

Notional Amount	$ 50,000,000
Trade Date	31 Dec. 2006
Effective Date	31 Dec. 2006
Maturity Date	31 Dec. 2016
Reference Year	2015
Fixed Rate	**0.8765% (or 87.65 basis points)**
Fixed Rate Payer	XYZ Investment Bank
Fixed Amount	Notional Amount $\times$ Fixed Rate $\times$ 100
Reference Rate	LifeMetrics Index
Floating Amount Payer	XYZ Pension or Annuity Provider
Floating Amount	Notional Amount $\times$ Reference Rate $\times$ 100
Settlement	Net settlement = Fixed amount - Floating amount

Based on our DEJD model and the q-forwards product structure above, the fixed rate can be calculated with the closed-form formula (12) directly.

$$
\begin{aligned}
E^*[\mu_t] &= \sum_x W_x \times \{\exp(a_x) \times E^*[\exp(b_x k_t)]\} \\
&= \sum_x W_x \times \{\exp(a_x + b_x k_0 + b_x t(\alpha^* - \frac{1}{2}\sigma^{*2} - \lambda^* \gamma^*) + \frac{1}{2} b_x^2 \sigma^{*2} t + \lambda^* t(\frac{p^* \eta_1^*}{\eta_1^* - b_x} + \frac{q^* \eta_2^*}{\eta_2^* + b_x} - 1))\}
\end{aligned}
$$

Table 6. Parameters for the Closed-Form Solution of q-Forward[2]

Age-specific Parameters	W_x	a_x	b_x
<1	0.013818	-3.4087	0.1455
1-4	0.055317	-6.2254	0.1960
5-14	0.145565	-7.1976	0.1492
15-24	0.138646	-6.2957	0.0994
25-34	0.135573	-5.9923	0.1044
35-44	0.162613	-5.4819	0.0855
45-54	0.134834	-4.7799	0.0608
55-64	0.087247	-4.0137	0.0468
65-74	0.066037	-3.2347	0.0426
75-84	0.044842	-2.4196	0.0409
>85	0.015508	-1.6119	0.0290

Other Parameters	Value
k_0	-10.302
t	10
α^*	-0.20
σ^*	0.31
λ^*	0.029
γ^*	-1.25
p^*	0.035
η_1^*	0.89
q^*	0.065
η_2^*	0.93

The fixed rate or equivalently the mortality forward rate quoted by an investment bank would be formed using a combination of

i. best estimate mortality projection

ii. a risk premium

iii. mid-to-bid spread

The best estimate of mortality will depend on the model used, e.g., the Lee-Carter model, and so may be biased with the model is not the right one. The risk premium must be appropriate for the market and based on transactions there; there are very few transactions in the mortality based derivatives market and so the risk premium calculation is problematic.

The calculation here uses the well know risk neutral valuation approach with adjustments for mortality and longevity jumps. The jump processes play a role in fitting the data and in estimating the risk premium. The mortality forward rate flows from the closed-form solution. For the U.S. data used here, the fixed rate 10 year q-forward contract is priced at 0.8765% (or 87.65 basis point). This DEJD pricing may differ from that of the Lee-Carter or Chen-Cox models.

[2] We apply the risk-neutral measure change on the positive jump severity η_1^* and negative jump severity η_2^*.

1. The best estimate mortality projection will be different calculated by the Lee-Carter model, Chen-Cox model and our DEJD model. The ignorance of the asymmetric jump effect will make difference in the .fixed rate pricing and the asymmetric jump diffusion has significant effect on the projected mortality time-series. We use the same data, whole age groups, both sex in the reference year for the U.S. national population, and the result for the Best Estimate mortality projection for q-forward fixed rate is 0.8583% with Lee-Carter model, 0.8594% with Chen-Cox model, and 0.8765% with DEJD model.

Lee carter model captures the significant positive jump (1918 flu) and moderate negative jump in the baseline Brownian Motion process, which caused a larger standard deviation and a positive biased mean for the baseline Brownian Motion process. If we set Lee-Carter model as the benchmark and make the comparison of the Chen-Cox model and DEJD model, we conclude as followings.

Compared to Lee-Carter model, Chen-Cox model considers the significant positive jump and moderate negative jump in the normal jump process, not in baseline BM process, which causes the standard deviation reduced and the mean reduced in the baseline BM process. However, in the projection of future mortality rate, the jumps are assumed to occur symmetrically in the jump diffusion. So the overall jump effect is offset by each other (positive and negative). The jump diffusion doesn't remarkably affect the mean of BM process. So the pricing of the fixed rate is not very significantly different from that with the Lee-Carter model.

Compared to Lee-Carter model, our DEJD model considers the significant positive jump and moderate negative jump in the asymmetric jump process, not in baseline BM process, which also causes the standard deviation reduced and the mean reduced in the baseline BM process. The difference from the Chen-Cox model is that, in the projection of the future mortality rate, the jumps are assumed to occur asymmetrically following the historical rule of infrequent significant positive jumps and frequent moderate negative jumps. In this way, the overall jump effect adds a positive increment to the mean of BM process, which causes the pricing of the fixed rate higher than that with the Lee-Carter model.

2. The difference of the pricing also comes from the different implied market price of risk. The implied market price of risk in our model is estimated by only one product, i.e., the Swiss Re

Mortality Catastrophe Bond, due to the lack of sufficiency of mortality securities in the market. In the previous literatures, the annuities and other actively traded mortality linked securities are employed to calibrate the implied market price of risk. Appling the different estimated implied market price of risk in the formula can cause the difference of the quoted q-forward fixed rate.

3. In the realistic deals, the mid-bid spread (defined as the half of the bid-ask spread), and other factors in the judgment call also cause the difference of the quoted fixed rate of q-forward.

5 Conclusion

This paper proposes a quantitative model to price the mortality-linked securities, and provide a possible approach to measure and manage longevity/mortality risk. The dramatic and dynamic improvement of the life expectancy has attracted public attention to the longevity risk. The inadequate management of longevity risk will seriously affect the asset and liability balance of the pension fund and annuity providers. On the other side, the life insurers have begun to concern the mortality risk, caused by sudden influenza or another catastrophic source. A series of mortality linked securities, e.g. longevity bond, mortality bond, and other types of securities have been issued to manage and transfer the risk. Recently, the life settlement securitization involving the life expectancy of insurance policy holders has boomed. Hence, modeling and pricing the mortality linked securities is crucial to the risk management, product innovation and formation of the liquid intermediate market.

Our model stresses the longevity jumps and the mortality jumps which are critical in the pricing, since the jumps are the extreme events and main resource of catastrophic risk which cannot be ignored. The lesson to be learned is that the lack of consideration of catastrophic risk in mortgage default rate has lead to the crash of MBS, CDO market and the meltdown of the whole financial market. The mortality jumps (like the 1918 flu) have the short-term intensified effect, while the longevity jumps (caused by the pharmaceutical or medical innovation) have the long-term gentle effect. As a result, we need the model to capture the different features of the longevity jumps and mortality jumps, separately.

We propose a stochastic mortality model to capture the observed feature of the historical mortality rate, and to price the mortality linked securities. The baseline component of the model incorporates the advantage of the Lee-Carter framework, which describes the main trend and regular dynamics of historical mortality rate, and also is able to adjust for the cohort effect. The jump diffusion component applies the compound poisson-Double Exponential Jump Diffusion to describe longevity jumps (negative jumps) and mortality jumps (positive jumps) separately. Our paper contributes to the existing literature by accommodating the different features of longevity jumps and mortality jumps. Hence the model fits the mortality time-series increment distribution much better than the previous models and explains the distribution skewness effect. Meanwhile, the model has the advantage of mathematical tractability and closed-form solution for the standard securities, like q-forward, whose price depends on the expected future mortality rate only. Since the DEJD model has the concise specification and closed-form density function, the likelihood function for the parameters can be easily expressed. In this way, the small number of parameters and concise likelihood function facilitates the calibration and application of the model in practice.

We calibrate the model with the historical mortality rate data 1900-2004 from National Center for Health Statistics. We apply the risk-neutral pricing method to derive the implied market price of risk, based on the Swiss Re Mortality Catastrophe Bond. In practical application, we choose the q-forward as an example for mortality linked security valuation. The q-forward is the standardized contracts and the basic building blocks for more complex securities. The price of the q-forward can be easily calculated by a closed-form formula, since the value of q-forward only depends on the expected future mortality rate. We compare the prices of the q-forward fixed rate, calculated by the Lee-Carter model, Chen-Cox model and our DEJD model separately. Compared with the benchmark Lee-Carter model, the price is lower from the Chen-Cox model and is higher from our model. The underlying reason is the Chen-Cox model employs the symmetric jump and our model employs the asymmetric jump. The infrequent significant mortality jump and frequent moderate longevity jump phenomenon in the historical data is appropriately reflected in our DEJD model, and affects the securities pricing through the best estimate mortality projection.

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