Stochastic Mortality Modeling with Lévy processes

Chou-Wen Wang[1]
Hong-Chih Huang[2]
I-Chien Liu[3]

Abstract

In the classical Lee-Carter model, the mortality indices k_t assumed to be a random walk model with drift is normally distributed. The assumption of normality is stacked against the hard fact: the mortality indices have tails thicker than those forms such as a normal distribution and appear to be skewed. As a result, we adopt two infinitely divisible distributions – Generalized Hyperbolic (GH) and Classical Tempered Stable (CTS) distributions–to model the mortality indices. Based on the ninety-five-year mortality data of ten countries from Human Mortality database (HMD), the criteria of Akaike Information Criterion, Bayes Information Criterion, Kolmogorov-Smirnov tests and mean absolute percentage errors in mortality projection consistently indicate a preference for the mortality indices with GH or CTS distributions over those with normal distribution.

Keywords: Stochastic Mortality Model; Generalized Hyperbolic Processes; Classical Tempered Stable Processes.

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[1] Associate Professor, Department of Risk Management and Insurance, National Kaohsiung First University of Science and Technology, Kaohsiung, Taiwan.
[2] Professor, Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan.
[3] Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan. E-mail: horn8802@gmail.com *Corresponding author.

Introduction

Recently, it is widely acceptable that mortality is a stochastic process. Traditional valuation methods relying on deterministic mortality models may contribute to mispricing problems. Mortality models make mortality risk management available to quantify the mortality and longevity risks and provide the foundation of pricing and reserving.

Among all stochastic mortality models, the Lee-Carter model, proposed by Lee and Carter in 1992, is one of the most popular choices because of ease of implementation and acceptable prediction errors in empirical studies from many countries. Various modifications of the Lee-Carter model have been extended by Brouhns, Denuit, and Vermunt (2002), Renshaw and Harberman (2003), Denuit, Devolder, and Goderniaux (2007), and further revisited by Li and Chan (2007) to attain a broader interpretation. Cairns, Blake, and Dowd (2006) propose a two-factor stochastic mortality model, denoted the CBD model, under which the first factor affects mortality at all ages, whereas the second factor affects mortality at higher ages much more than at lower ages. Furthermore, Cairns et al. (2009) class and compare eight stochastic mortality models, including extension of CBD model, in England and Wales and in the United States. Besides, Haberman and Renshaw (2009) enhance Lee-Carter model approach to forecast age-period-cohort mortality and test for robustness. However, short-term catastrophe mortality shocks, such as influenza pandemic in 1918 and the tsunami in December 2004, may lead to a much higher (or lower) mortality rates. The above papers do not include mortality jumps.

To take into account the mortality jumps, Biffis (2005) use affine jump-diffusions to model asset prices and mortality dynamics, addressing the risk analysis and market valuation of life insurance contracts. Luciano and Vigna (2005) find that, for Italian mortality data, introducing a jump component provides better fit than a diffusion component in the stochastic mortality processes. Cox, Lin and Wang (2006) combine a geometric Brownian motion and a compound Poisson process to model the age adjusted mortality rates for US and UK, evaluating the first pure mortality security, Swiss Re bond. In addition, Lin and Cox (2008) combine a Brownian motion and a discrete Markov chain with log-normal jump size distribution for pricing mortality-based securities in an incomplete market framework. Chen and Cox (2009) incorporate a jump process into the Lee-Carter model, and use it to forecast mortality rates and analyze mortality securitization. Therefore, the above papers use diffusion processes with jump components, finite activity Lévy processes, to describe the dynamics of morality rates. However, except finite-activity Lévy processes, non-normal innovations can be also generated by a pure jump Lévy process. Recent studies indicate that asset returns can be modeled as pure jump processes (Aït-Sahalia, 2001; Geman et al., 2001; Geman, 2002). Carr et al. (2002) demonstrate empirically that a diffusion component is unnecessary if the pure jump process allows for infinite activity around the origin. Our work aims to model human mortality in a similar way: as in Chen and Cox (2009), we consider the stochastic mortality model driven by pure jump processes.

A pure jump Lévy process can display either finite activity or infinite activity. The classical example of a finite-activity jump process is the compound Poisson jump process. Examples of infinite-activity jump process include the normal inverse Gaussian (NIG) model of Barndorff-Nielsen (1998), the generalized hyperbolic (GH) model of Eberlein et al. (1995), the variance gamma (VG) model of Madan and Milne (1991) and Madan et al. (1998) and the Classical Tempered Stable (CTS) distribution of Rosiński (2007). Among these, the GH model includes the NIG model as a special case and the CTS model includes VG as a special case. Therefore, we focus on the GH model and the CTS model, and report on the goodness of fit and mortality projection obtained on the mortality rates.

To our knowledge, Hainaut and Devolder (2008) first applies α-stable subordinators—infinite-activity strictly positive Lévy processes—to model the mortality hazard rates. However, in the Lee-Carter model, the first difference of mortality indices may be negative to reflect the mortality improvement. As a

Consequence, unlike Hainaut and Devolder (2008), in this paper we incorporate the GH and CTS processes into the original Lee–Carter model, and use it to fit and forecast the mortality rates. We select the ninety-five-year mortality data of ten countries—Demark, Finland, France, Iceland, Italy, Netherlands, Norway, Spain, Sweden and Switzerland—as the observed data. We fit the model to the mortality rates from 1912 to 2001 and forecast the development of the mortality curve for the consequent five years. According to the Jarque-Bera (JB) test statistics, the assumptions of normality of mortality indices are almost rejected. In addition, based on Akaike Information Criterion (AIC), Bayes Information Criterion (BIC), Kolmogorov-Smirnov (KS) tests and mean absolute percentage errors (MAPE) in mortality projection, the empirical results consistently indicate that the GH and CTS distributions are better than the normal distribution in modeling the mortality indices.

The remainder of this paper is organized as follows. In Section 2, the Lee-Carter model with GH and CTS innovations is illustrated. The dynamics of mortality indices are also provided. Section 3 empirically tests the good of fit of stochastic mortality models with GH, CTS and normal innovations. The mortality projection is also studied. The last section draws conclusions about our findings.

The Lee-Carter Model with GH and CTS Innovations

In this section, we first review the classical Lee-Carter model under which the mortality index follows an ARIMA model with normal innovation. Based on the mortality data of ten countries, however, most of the mortality indices exhibit non-zero skewness and excess kurtosis. As a result, we use the GH and CTS distributions to model the dynamics of mortality indices.

The Lee-Carter model

We analyze the changes in mortality as a function of both age *x* and time *t*. Used for mortality forecasting is the classical Lee-Carter model, namely,

$$ln\left(m_{x,t}\right) = \alpha_x + \beta_x k_t + e_{x,t}, \tag{1}$$

where $m_{x,t}$ is the central death rate for age *x* in calendar year t which is defined as running from time t to time $t+1$. This structure is designed to capture age-period effects; α_x describe the average pattern of mortality over age group; β_x represents the age-specific patterns of mortality change, indicating the sensitivity of the logarithm of the force of mortality at age *x* to variations in the time index k_t; k_t explains the time trend of the general mortality level; and $e_{x,t}$ represents the deviation of the model from the observed log-central death rates, which is expected to be a normal distribution with zero mean and a small variance relatively (Lee, 2000).

We use the approximation method to fit the three parameters. According to two constraint conditions $\sum_t k_t = 0$ and $\sum_x \beta_x = 1$, $\hat{\alpha}_x$ is simply the average value over time of $ln\left(m_{x,t}\right)$, and $\hat{k}_t$ is the sum over various ages of $\log\left(m_{xt}\right) - \hat{\alpha}_x$. Using $ln\left(m_{x,t}\right) - \hat{\alpha}_x$ as the dependent variable and $\hat{k}_t$ as the explanatory variable, we can obtain $\hat{\beta}_x$ by using a simple regression model without intercept parameter.

To forecast future dynamics of mortality, Lee and Carter (1992) assume that the α_x and β_x remain constant over time and forecast the future dynamics of mortality index k_t using an ARIMA(0,1,0) model as follows:

$$k_t - k_{t-1} = \gamma + \varepsilon_t, \tag{2}$$

where γ is a drift; and ε_t is a sequence of independent and identically Gaussian random variables with mean 0 and variance σ^2.

Normality Test for Mortality Indices

In this subsection, the mortality indices k_t of ten countries for male and female are depicted in Figure 1. Figure 1 provides evidence of mortality improvement. In addition, as argued by Chen and Cox (2009), these curves even jump at the same time, which is remarkably evidenced around the years 1918 and 1940 because of 1918 influenza pandemic, World War I and World War II. The similar patterns of these curves suggest that we should not exclude these events when modeling the mortality indices.

[Insert Figure 1 here]

To further examine the assumption of normality of mortality indices in the Lee-Carter model, we use the JB test statistic (Jarque and Bera, 1980), a goodness-of-fit measure of departure from normality, as follows:

$$JB = n\left[\frac{s^2}{6} + \frac{(k-3)^2}{24}\right] \tag{3}$$

where n is the sample size; s is the sample skewness; and k is the sample kurtosis. Table 1 exhibits the JB test results, together with the skewness and excess kurtosis, for the first difference of male (female) mortality indices of ten countries. Clearly, except for mortality indices of Demark female, Finland female and Island male and female, they follow asymmetric leptokurtic distributions with peak around the mean and fatter tails. The assumption of normality is inadequate for the mortality indices. As a result, we adapt the pure jump Lévy processes—GH and CTS processes— to model the non-normal property of mortality indices.

[Insert Table 1 here]

Modeling Mortality indices as GH and CTS Processes

In this section, we use two Lévy processes—GH and CTS processes—to model the time trend of k_t. Lévy process is a continuous time stochastic process with stationary independent increments. For two common examples, one is Brownian motion, the purely continuous Lévy process, and another is compound Poisson process. Bertoin (1996) proposes that the following Lévy-Khinchin representation states that a characteristic function $\phi(\omega)$ of a pure jump Lévy process $\{L(t); t \geq 0\}$ can be written as

$$\phi_t(\omega) \equiv E_Q\left(\exp\left(i\omega L(t)\right)\right) = \exp\left(-t\psi(\omega)\right),\ \forall\, \omega \in R, \qquad (4)$$

where $\psi(\omega)$, called characteristic exponent, is given by

$$\psi(\omega) = \int_{R-\{0\}} \left(1 - e^{i\omega x} + i\omega x 1_{|x|<1}\right) v(dx), \qquad (5)$$

where the Lévy measure v uniquely specifies the Lévy process. For a infinite-activity pure jump Lévy process, its Lévy measure satisfies the following conditions:

$$\int_{-\infty}^{\infty} v(dx) = \infty,\ \int_{-\infty}^{\infty} 1(|x| \geq 1) v(dx) < \infty \text{ and } \int_{R-\{0\}} x^2 1(|x|<1) v(dx) < \infty. \qquad (6)$$

The first and second conditions mean that the process has infinite number of small jumps but finite number of large jumps. The third condition states that the Lévy measure must be square-integrable around the origin. Consequently, we can use the infinite-activity pure jump Lévy processes to capture the dynamics of mortality indices.

Because the mortality indices are non-normal distribution, in this paper we assume that the first difference of mortality index follows an infinite-activity pure jump Lévy process. Therefore, Equation (2) can be rewritten as follows:

$$k_t - k_{t-1} = L_1, \tag{7}$$

where L_1 is the Lévy process with unit time scale. When L_t is the GH (CTS) process, the mortality index k_t follows GH (CTS) distribution. The characteristic functions of GH and CTS distributions are listed in Table 2. process Lévy processes in Table 2. K is the modified Bessel function of the second kind and Γ is gamma function.

[Insert Table 2 here]

The GH distribution includes many interesting distributions as special and limiting cases including the NIG distribution, the hyperbolic distribution, the normal distribution, Student *t* distribution, the skew hyperbolic-*t* distribution, and the variance gamma (VG) distribution. The Classical Tempered Stable distribution includes many distributions as special cases, for example, the truncated Lévy flight (Koponen, 1995) is obtained if $\lambda_+ = \lambda_-$, the CGMY distribution (Carr et al., 2002) is obtained if $C_+ = C_-$, and Variance Gamma distribution (Madan et al., 1998) is obtained if $\alpha = 0$ and $C_+ = C_-$. Therefore, it provides a more flexible and abundant mortality index models with GH and CTS innovations than that with normal innovation.

Empirical Analysis

In this section, we first illustrate the mortality data and then investigate the good-of-fit of mortality index models with GH, CTS and normal distributions. Using their estimated parameters, we also project the mortality rates for each sex in ten countries.

Mortality Data

The models presented in this paper are fitted to the mortality rates of Demark, Finland, France, Iceland, Italy, Netherlands, Norway, Spain, Sweden and Switzerland. The sample period is from 1912 to 2006 with ages ranging from 0 to 100 obtained from Human Mortality Database (HMD) website[1]. Using the mortality date from 1912 to 2001, we fit the first difference of k_t with three distributions−GH, CTS and normal distributions, and then predict the next five-year mortality rates.

Model Comparison

For the sake of comparison, we use log likelihood function (LLF), Akaike Information Criterion (Akaike, 1974), Bayesian information criteria (Schwarz, 1978), and KS test as good-of-fit measures. The AIC is defined as

$$\text{AIC} = (-2*\text{LLF}) + (2*\text{NPS}), \tag{8}$$

where NPS is the effective number of parameters being estimated. The BIC is defined

[1] http://www.mortality.org/

as

$$\text{BIC} = (-2*\text{LLF}) + (\text{NPS} * \log(\text{NOS})), \tag{9}$$

where NOS is the number of observations. In these criteria, a higher value of LLF and a smaller value of AIC and BIC mean a better good-of-fit mortality model. In addition, the null hypothesis of KS test (Kolmogorov, 1933) is $H_0 : G(x) = F^*(x;\theta)$ for all sample data x and the known parameters θ of the distribution, where $G(x)$ represents the empirical distribution function of sample mortality index and $F^*(x;\theta)$ represents the hypothesized cumulative distribution function. The test statistic is defined as

$$KS = \sup_{\{x\}} \left| F^*(x;\theta) - G(x) \right|. \tag{10}$$

A higher p-value of KS test means a better good-of-fit mortality model.

Table 3 and Table 4 show that the results are fitted by GH, CTS and normal distributions. Based on the criteria of LLF, AIC and BIC, most of the results indicate that mortality index model with GH or CTS innovations provide a better good-of-fit than that with normal innovation. For the Iceland morality data, it is not surprise that the criteria indicate a preference for the model with normal innovation over those with GH and CTS innovation since the JB test do not reject the assumption of normality. In addition, we find that the KS p-value also prefer the models with GH or CTS innovations to that with normal innovation.

[Insert Table 3 here]

[Insert Table 4 here]

Next, we also compare their performance of mortality projection. Based on the parameters estimated from the time period 1912-2001, Table 5 and Table 6 show that the percentile of Mean Absolute Percentage Error (MAPE) for 2002-2006. The definition of the MAPE is calculated as follows:

$$MAPE = 100\% \times \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_i - F_i}{A_i}\right|, \tag{11}$$

where A_i is historical mortality rate and F_i is the forecast mortality rate. We simulate 100,000 paths to test the performance of the prediction. For each simulation path, we obtain a corresponding MAPE and then sort these MAPE to get the percentile. The 5, 10, 90 and 95 percentiles, together with the medium, are obtained in Table 5 for female and Table 6 for male. Table 5 and Table 6 demonstrate that except for Denmark female and each sex of Iceland, the MAPE criterion indicates that the models with GH or CTS innovations provide better prediction ability than that with normal distribution. Therefore, according to AIC, BIC, KS test and MAPE criteria, we reach a conclusion that the mortality index models with GH or CTS innovations provide a better good-of-fits than that with normal innovation.

[Insert Table 5 here]

[Insert Table 6 here]

Conclusions

Recently, many researchers have been examining mortality rates and exploring models. Some studies believe that the improvement of mortality rates has jump property. In this paper, we make the first attempt to incorporate two infinity-activity pure jump Lévy processes - Generalized Hyperbolic (GH) and Classical Tempered Stable (CTS) Processes - into Lee-Carter model respectively.

Based on the ninety-five-year mortality data of each sex of ten countries from Human Mortality database (HMD), the criteria of Akaike Information Criterion, Bayes Information Criterion, Kolmogorov-Smirnov tests and mean absolute percentage errors in mortality projection consistently indicate a preference for the mortality indices with GH or CTS distributions over those with normal distribution. Therefore, we suggest that, when the Lee-Carter model is applied, it provide a better good-of-fits by assuming that the mortality indices follow the GH or CTS processes. In this paper, we do not provide the valuation and hedging of mortality securities. Using the Esscher transform, future research may focus on how to price and hedge the mortality-linked security such as Swiss Re mortality bond issued in 2003.

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Table 1 JB Test, Skewness and Excess Kurtosis of the First Difference of k_t

Country	Gender	P-value	Skewness	Excess Kurtosis
Denmark	Female	0.248	-0.302	0.426
	Male	<0.001	-0.738	2.783
Finland	Female	0.284	0.352	0.063
	Male	<0.001	1.476	13.001
France	Female	<0.001	-0.219	6.612
	Male	<0.001	0.462	8.203
Iceland	Female	0.083	0.327	0.776
	Male	0.063	0.025	1.095
Italy	Female	<0.001	1.418	27.125
	Male	<0.001	-0.120	9.555
Netherlands	Female	<0.001	-1.670	12.193
	Male	<0.001	-3.037	23.809
Norway	Female	<0.001	0.904	6.299
	Male	<0.001	0.126	4.906
Spain	Female	<0.001	1.312	17.559
	Male	<0.001	0.962	10.622
Sweden	Female	<0.001	1.354	11.542
	Male	<0.001	1.815	17.930
Switzerland	Female	<0.001	1.049	12.520
	Male	<0.001	0.615	19.881

Table 2 Characteristic Functions of GH and CTS Distributions

Processes	Characteristic Function $\phi(\omega)$
Generalized Hyperbolic model $GH(\alpha,\beta,\delta,\lambda,\mu)$	$e^{i\mu u}\left(\frac{\alpha^2-\beta^2}{\alpha^2-(\beta+iu)^2}\right)^{\lambda/2}\frac{K_\lambda\left(\delta\sqrt{\lambda^2-(\beta+iu)^2}\right)}{K_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)}$
Classical Tempered Stable $CTS(\alpha,C_+,C_-,\lambda_+,\lambda_-,m)$	$\exp\left\{ium+\Gamma(1-\alpha)\left(C_+\lambda_+^{\alpha-1}-C_-\lambda_-^{\alpha-1}\right)\right.$ $+C_+\Gamma(-\alpha)\left((\lambda_+-iu)^\alpha-\lambda_+^\alpha\right)$ $\left.+C_-\Gamma(-\alpha)\left((\lambda_-+iu)^\alpha-\lambda_-^\alpha\right)\right\}$

Note: K is the modified Bessel function of the second kind, and Γ is a gamma function.

Table 3 Good-of-Fit Measures for Female

Female		LLF	AIC	BIC	KS p-value
Denmark	Normal	-281.6312	567.2624	572.2620	0.0143
	GH	-266.0407	542.0813	554.5804	**0.9954**
	CTS	**-266.0381**	**542.0762**	**554.5752**	0.9942
Finland	Normal	-317.3149	638.6298	643.6294	0.0175
	GH	**-292.8436**	**595.6872**	**608.1862**	**0.6544**
	CTS	-292.8464	595.6929	608.1919	0.6493
France	Normal	-311.0207	626.0413	631.0410	0.0027
	GH	**-289.2394**	**588.4787**	**600.9778**	**0.9225**
	CTS	-291.3043	592.6087	605.1077	0.6013
Iceland	Normal	**-323.9160**	**651.8320**	**656.8316**	0.5365
	GH	-362.3137	734.6274	747.1264	**0.9489**
	CTS	-362.3246	734.6492	747.1483	0.9385
Italy	Normal	-319.6590	643.3180	648.3176	0.0002
	GH	**-284.8698**	**579.7396**	**592.2387**	**0.8639**
	CTS	-287.3213	584.6427	597.1417	0.3910
Netherlands	Normal	-314.2988	632.5976	637.5973	0.0013
	GH	**-293.1537**	**596.3074**	**608.8064**	**0.9261**
	CTS	-293.5052	597.0104	609.5095	0.7154
Norway	Normal	-272.4171	548.8343	553.8339	0.1826
	GH	**-265.6993**	**541.3985**	**553.8976**	**0.9990**
	CTS	-265.7121	541.4241	553.9232	0.9984
Spain	Normal	-307.0227	618.0454	623.0451	0.0058
	GH	**-278.6876**	**567.3751**	**579.8742**	0.8670
	CTS	-279.8252	569.6504	582.1494	**0.9079**
Sweden	Normal	-300.1478	604.2955	609.2951	0.0830
	GH	**-271.8271**	**553.6542**	**566.1533**	0.8476
	CTS	-273.0809	556.1618	568.6609	**0.9467**
Switzerland	Normal	-301.6165	607.2331	612.2327	0.1811
	GH	**-279.7489**	**569.4978**	**581.9968**	**0.9985**
	CTS	-279.8568	569.7137	582.2127	0.9929

Table 4 Good-of-Fit Measures for Male

Male		LLF	AIC	BIC	KS p-value
Denmark	Normal	-259.6505	523.3010	528.3006	0.0425
	GH	-253.6852	517.3704	529.8695	0.7456
	CTS	**-253.1297**	**516.2595**	**528.7585**	**0.7683**
Finland	Normal	-322.9167	649.8334	654.8330	0.0012
	GH	**-312.1082**	**634.2164**	**646.7155**	**0.9880**
	CTS	-313.7963	637.5927	650.0917	0.6842
France	Normal	-323.1906	650.3813	655.3809	0.0000
	GH	-297.9508	605.9016	618.4006	**0.9966**
	CTS	**-291.7267**	**593.4535**	**605.9525**	0.2486
Iceland	Normal	-323.3071	650.6143	655.6139	0.5617
	GH	**-342.3913**	**694.7827**	**707.2817**	**0.9946**
	CTS	-342.4289	694.8578	707.3568	0.9931
Italy	Normal	-320.0279	644.0558	649.0554	0.0022
	GH	-301.0792	612.1584	624.6575	0.8997
	CTS	**-286.8498**	**583.6996**	**596.1986**	**0.9018**
Netherlands	Normal	-322.3932	648.7863	653.7859	0.0002
	GH	**-293.6083**	**597.2166**	**609.7157**	**0.8486**
	CTS	-295.6467	601.2934	613.7925	0.8130
Norway	Normal	-270.8644	545.7288	550.7284	0.0022
	GH	**-261.6509**	**533.3017**	**545.8008**	**0.9998**
	CTS	-262.0067	534.0134	546.5124	0.9880
Spain	Normal	-309.9867	623.9735	628.9731	0.0039
	GH	**-286.4151**	**582.8302**	**595.3292**	**0.9921**
	CTS	-286.9939	583.9879	596.4869	0.8771
Sweden	Normal	-275.6614	555.3228	560.3224	0.0166
	GH	**-257.7139**	**525.4279**	**537.9269**	**0.8817**
	CTS	-258.2076	526.4151	538.9141	0.6671
Switzerland	Normal	-303.0029	610.0058	615.0054	0.0531
	GH	**-275.4430**	**560.8861**	**573.3851**	**0.9761**
	CTS	-276.5020	563.0040	575.5030	0.8912

Table 5 Percentile of MAPE of Mortality Projection for Female (Unit: %)

Female		5%	10%	Medium	90%	95%
Denmark	Normal	**3.4661**	**3.4848**	**3.5996**	**3.9596**	**4.0840**
	GH	3.4821	3.5034	3.7137	4.4009	4.6574
	CTS	3.4778	3.5005	3.7161	4.2933	4.5534
Finland	Normal	7.1603	7.2002	7.6322	8.5164	8.8146
	GH	**7.1345**	**7.1602**	**7.4194**	**8.0261**	**8.2470**
	CTS	7.1348	7.1723	7.4431	8.0841	8.3508
France	Normal	3.5614	3.5894	3.8911	5.0650	5.4992
	GH	**3.5217**	**3.5361**	**3.7007**	**4.8471**	**5.4722**
	CTS	3.5319	3.5536	3.7987	5.3770	6.3538
Iceland	Normal	10.5264	10.5812	11.0350	12.5004	13.0243
	GH	10.5170	**10.5588**	11.0394	12.6986	13.5475
	CTS	**10.5136**	10.5598	**10.9045**	**12.4920**	**13.2356**
Italy	Normal	4.1625	4.1903	4.5423	6.0183	6.6042
	GH	**4.1252**	**4.1372**	**4.2982**	**5.3761**	**5.9318**
	CTS	4.1316	4.1423	4.3559	5.8481	6.7074
Netherlands	Normal	2.9293	2.9699	3.3566	4.6307	**5.0872**
	GH	**2.8961**	**2.9166**	**3.1286**	**4.5533**	5.7357
	CTS	2.9043	2.9279	3.2173	4.7998	5.8568
Norway	Normal	**5.4941**	**5.5092**	5.5938	**5.8619**	**5.9884**
	GH	5.4961	5.5112	5.5928	5.8806	6.0660
	CTS	5.5019	5.5119	**5.5906**	5.8833	6.0096
Spain	Normal	4.2810	4.3192	4.5913	5.4238	5.8462
	GH	**4.2544**	**4.2728**	**4.4257**	**5.1299**	**5.7671**
	CTS	4.2554	4.2823	4.4859	5.4550	6.0277
Sweden	Normal	3.8128	3.8275	3.9756	**4.4927**	**4.6687**
	GH	**3.7951**	**3.8049**	3.9123	4.5567	4.9596
	CTS	3.7963	3.8056	**3.9106**	4.5457	4.8950
Switzerland	Normal	4.0601	4.0793	4.2413	4.8233	**5.0412**
	GH	**4.0472**	**4.0574**	4.1814	**4.7487**	5.1421
	CTS	4.0487	4.0617	**4.1806**	4.7575	5.2766

Table 6 Percentile of MAPE of Mortality Projection for Male (Unit: %)

Male		5%	10%	Medium	90%	95%
Denmark	Normal	4.8640	4.8773	5.0096	**5.3456**	**5.4648**
	GH	4.8661	4.8789	4.9964	5.3920	5.5433
	CTS	**4.8629**	**4.8761**	**4.9925**	5.4102	5.6029
Finland	Normal	8.0899	8.1501	8.6171	10.3698	11.1702
	GH	8.0403	**8.0758**	**8.3922**	**9.9201**	11.3050
	CTS	**8.0382**	8.0908	8.4744	9.9820	**11.0628**
France	Normal	5.0559	5.1106	5.7243	7.9078	8.7535
	GH	**4.9889**	**5.0164**	**5.3147**	**6.8770**	**8.0444**
	CTS	4.9996	5.0433	5.5282	8.2538	9.3840
Iceland	Normal	11.2200	11.2890	11.6577	**12.8096**	**13.4348**
	GH	**11.2156**	11.2708	**11.6262**	12.8255	13.5292
	CTS	11.2196	**11.2807**	11.6611	12.9423	13.6455
Italy	Normal	5.2765	5.3187	5.7539	7.2911	**7.8877**
	GH	**5.2503**	**5.2719**	**5.5599**	7.4513	8.4238
	CTS	5.2513	5.2732	5.6115	**7.1392**	8.0599
Netherlands	Normal	4.0499	4.1410	4.8670	6.7219	7.3716
	GH	**3.9520**	**3.9871**	**4.3069**	**6.0929**	**7.3205**
	CTS	3.9620	4.0003	4.4576	6.8600	8.5175
Norway	Normal	6.4322	6.4506	6.5525	6.7766	**6.8710**
	GH	**6.4361**	**6.4458**	**6.5123**	**6.7655**	7.0372
	CTS	6.4327	6.4460	6.5198	6.8094	7.0229
Spain	Normal	3.7542	3.7951	4.1770	5.3553	**5.7795**
	GH	**3.7197**	**3.7376**	**3.9743**	**5.3256**	6.4212
	CTS	3.7199	3.7417	3.9897	5.4643	6.1843
Sweden	Normal	5.1920	5.2041	5.2909	5.6212	5.7940
	GH	**5.1787**	**5.1858**	**5.2427**	**5.5158**	**5.7149**
	CTS	5.1810	5.1889	5.2590	5.6478	5.9990
Switzerland	Normal	5.4998	5.5220	5.7623	6.5570	6.8685
	GH	**5.4769**	**5.4857**	**5.5864**	**6.1664**	**6.5336**
	CTS	5.4783	5.4886	5.6292	6.4505	7.2256

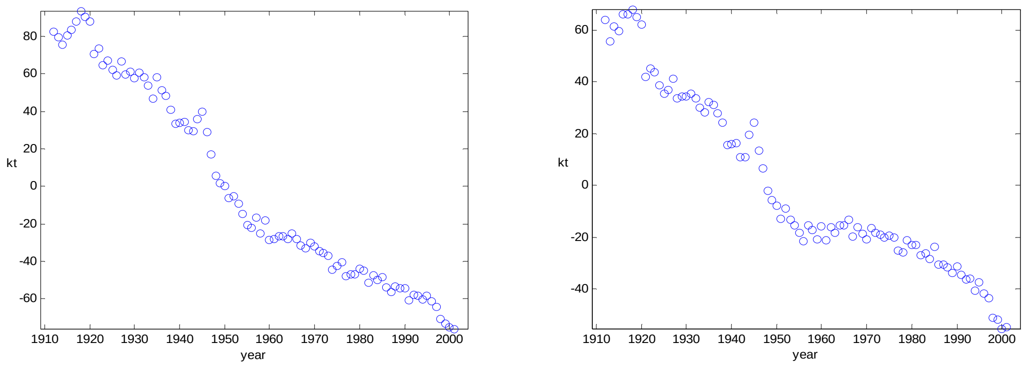

Figure 1A. Demark Female (Left) and Male (Right).

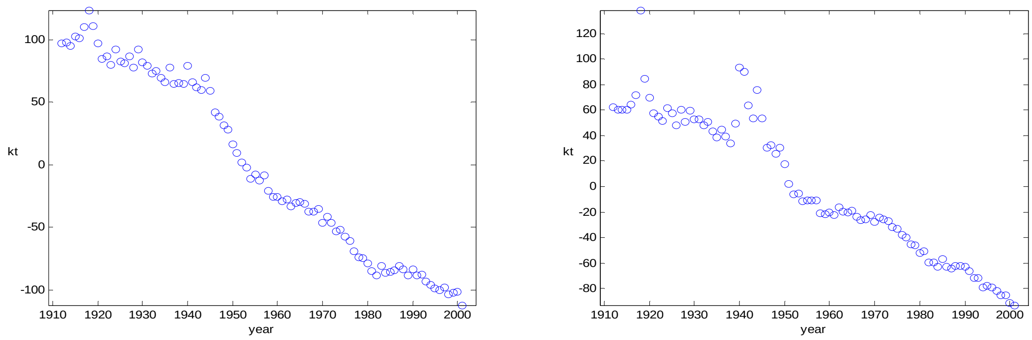

Figure 1B. Finland Female (Left) and Male (Right).

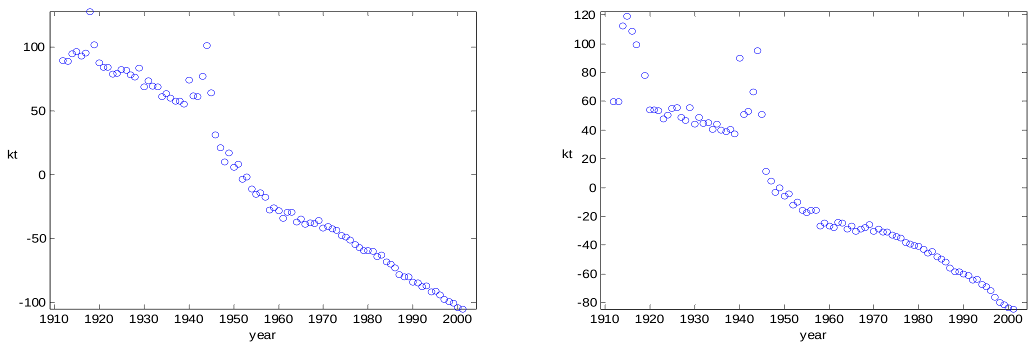

Figure 1C. France Female (Left) and Male (Right).

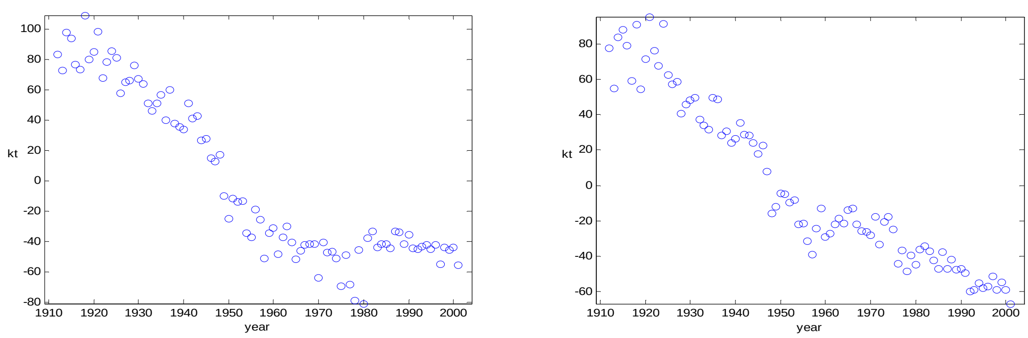

Figure 1D. Iceland Female (Left) and Male (Right).

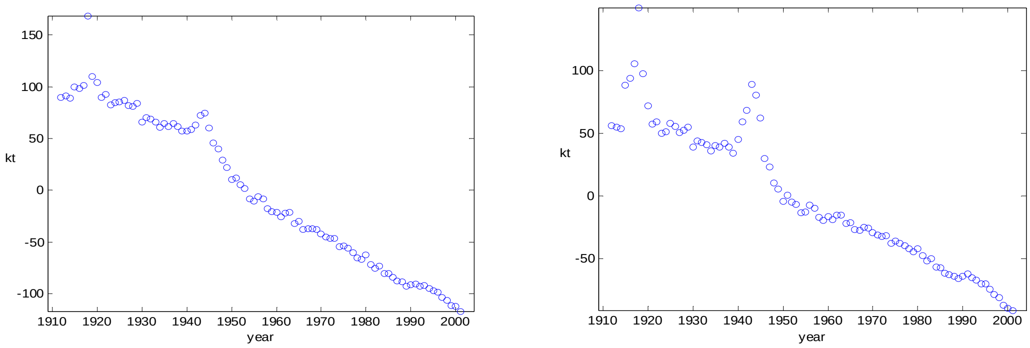

Figure 1E. Italy Female (Left) and Male (Right).

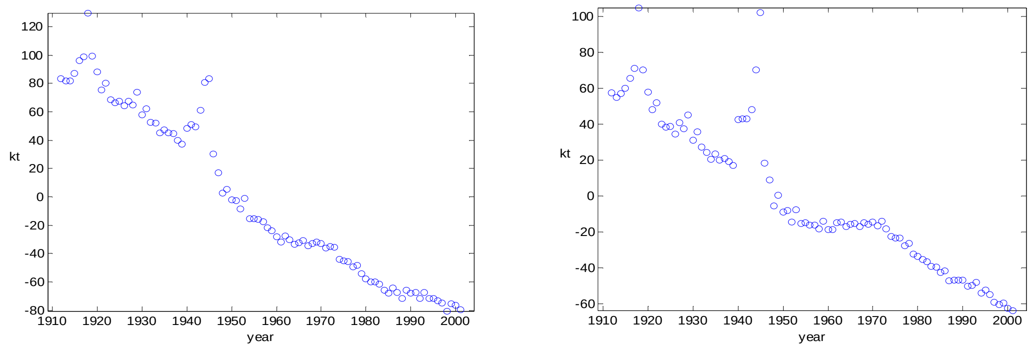

Figure 1F. Netherlands Female (Left) and Male (Right).

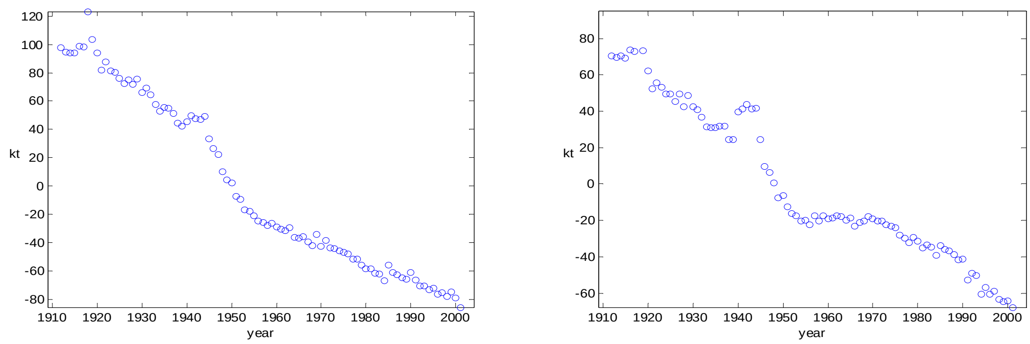

Figure 1G. Female (Left) and Male (Right).

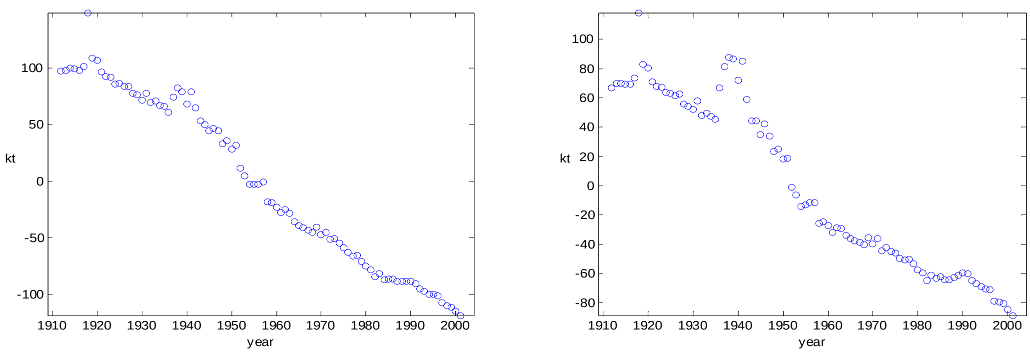

Figure 1H. Spain Female (Left) and Male (Right).

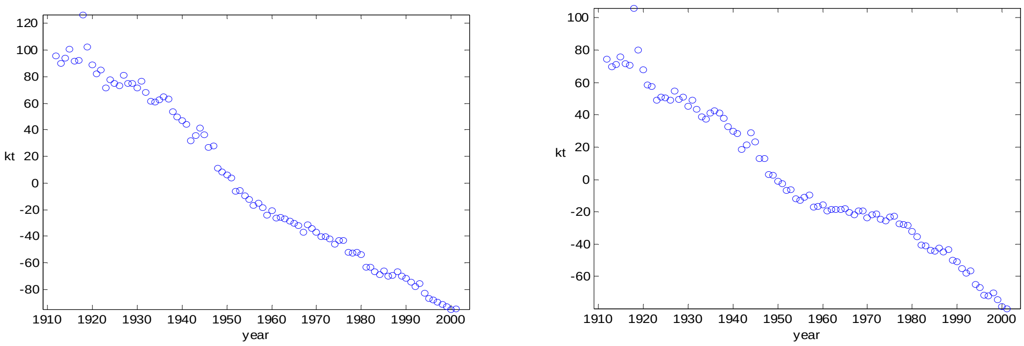

Figure 1I. Sweden Female (Left) and Male (Right).

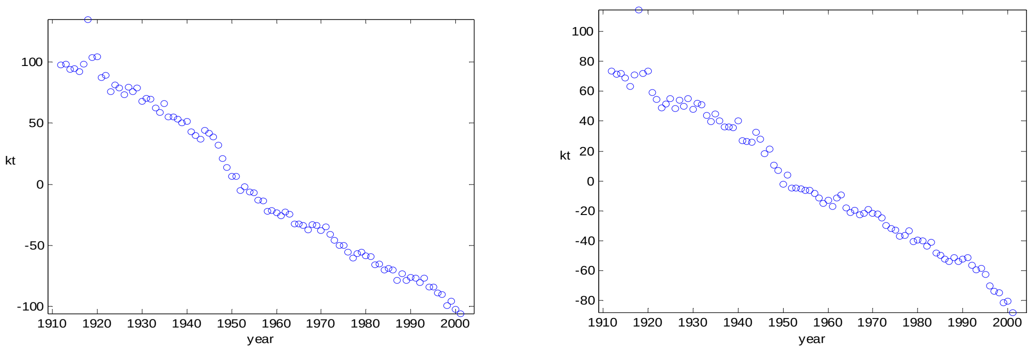

Figure 1J. Switzerland Female (Left) and Male (Right).