Securitization of Crossover Risk in Reverse Mortgages

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ABSTRACT

When the outstanding balance exceeds the housing value before the loan is settled, the insurer suffers an exposure to crossover risk induced by three risk factors: interest rates, house prices and mortality rates. Under the consideration of housing price risk, interest rate risk and longevity risk, we provide a three-dimensional lattice method which simultaneously captures the evolution of housing price and short-term interest rate to numerically calculate the fair valuation of reverse mortgages. For a mortgage reverse insurer, the premium structure of reverse mortgage insurance is determined by setting the present value of total expected claim losses equal to the present value of the premium charges. However, when the actual loss is higher than the expected loss, the insurer will incur an unexpected loss. To offset the potential loss, we also design a crossover bond to transfer the unexpected loss into the bond investors. Therefore, through the crossover bonds, the reverse mortgage insurers can partly transfer the crossover risk into the bond holders.

Keyword: reverse mortgages, crossover risk, longevity risk, crossover bonds.

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1. Introduction

Demographic aging offers one of the most serious challenges for the developed and developing countries. The trends of mortality improvement continue to threaten the security of social security systems worldwide. The dependent ratio, especially that of the aged—defined as the ratio of the number of senior dependents (over 65 years of age) to the total population (aged 15–64 years)—keeps rising in most countries, which means that the overall economy faces a greater burden to support an aging population.

Governments and industries seek to decrease their financial burden by deferring the retirement age and/or reducing the benefits people receive in a defined benefit pension plan. In addition, people might attain financial security in retirement through savings, purchasing private insurance, or continuing to work. However, for many people who earn less from their employment, the alterative is unrealistic and impractical. As demographic shifts increase the proportion of elderly homeowners who are often house-rich and cash-poor, reverse mortgages might provide a more reasonable solution for the aging homeowner population. Reverse mortgages are financial contracts that allow retirees to convert their home equities into either a lump sum or annuity income but still maintain ownership and residence until they die, sell or vacate their homes to live elsewhere.

There are three basic payment forms for reverse mortgages: (1) tenure, (2) term, or (3) line of credit. The distinguishing characteristic of the tenure mortgage is that it provides a monthly payment to the borrower as the borrower occupies the house. In contrast, a term loan provides monthly payments for only a fixed period. The line of credit permits the borrower to make draws at any time up to some maximum prespecified amount; the mortgage is not due and payable until the borrower sells the property, moves out permanently, or dies. At the mortgage's due date, the loan gets repaid with accumulated interest through the sale of the property. Furthermore, the lender can only receive the minimum of the entire debt or the net value of the property, which prevents the borrower from owing more than the value of the property. This nonrecourse clause makes the reverse mortgage difficult to price.

The reverse mortgage contracts involve a range of risks from the insurer's perspective. The outstanding balance usually accumulates at a faster rate than the appreciating rate of the housing value; therefore, if the outstanding balance exceeds the housing value before the loan is settled, the lender starts to incur a loss－namely crossover risk, one of the crucial risks to manage in reverse mortgages. Apparently, this crossover risk is induced by three risk factors: interest rates, house prices and mortality rates. As pointed out by Phillips and Gwin (1992), the lending feature of reverse mortgages subject loan providers to multiple risks. An increase of the lifespan of the loan resulted from mortality improvement or reduced mobility rates will impose a higher crossover risk. A rise in interest rates will speed up the rate at which the loan accumulates, and will possibly hit the crossover point earlier. Besides, a depressed real estate market will worsen the value of the home.

Szymanoski (1994) analyzes the risks involved with reverse mortgage insurance and explains a pricing model developed for the Home Equity Conversion Mortgage (HECM)—a publicly guaranteed reverse mortgage offered by the U.S. Department of Housing and Urban Development. Chinloy and Megbolugbe (1994) develop an alternative pricing model for a reverse mortgage in which the borrower receives payments as either a lump sum or an annuity. Both studies investigate reverse mortgages with a constant interest rate assumption. Boehm and Ehrhardt (1994) provided analysis of the risks associated with reverse mortgage loans and presented pricing models for reverse mortgages under interest rate risk inherent to fixed-rate reverse mortgages. They find that the interest rate risk of a reverse mortgage is greater than that of either a typical coupon bond or a regular mortgage. Mitchell and Piggott (2004) also explore the feasibility of developing the reverse mortgage market in Japan and conclude that it has the potential to relieve the fiscal burden on traditional, state-funded retirement provisions.

Rodda, Lam, and Youn (2004) analyzed the HECM program using a simulation model in which interest rates and house prices vary according to historically accurate transition probabilities followed by the 1-year Treasury since April 1953 and the OFHEO house price index since 1975. Ma, Kim and Lew (2007) analyze the risk of government insured reverse mortgages in Korea, using the Monte-Carlo simulation method considering the housing prices following geometric Brownian motion and interest rates following Vasicek process. Employing the Lee-Carter model with permanent jump effects and an ARIMA-GARCH housing pricing model, Chen, Cox and Wang (2010) price the non-recourse provision of reverse mortgages and compare it with calculated mortgage insurance premiums.

The loan balance of reverse mortgage may grow to exceed the property value at the time of termination because of multiple risks: termination risk (longevity risk), interest rate risk and housing price risk. However, most of the existing literature on risk modeling in the HECM program does not simultaneously consider the dynamics of mortality rates, interest rate and housing price. Therefore, under the consideration of housing price risk, interest rate risk and longevity risk, we price reverse mortgages with up-to-date methods. Specifically, we consider a reverse mortgage structured like an HECM scheme and assume that housing prices follow a Geometric Brownian motion (GBM) process (Cunningham and Hendershott, 1984; Kau, J. B., et al., 1992; Chinloy and Megbolugbe, 1994; Hilliard, J. E., et al., 1998; Szymanoski, 1994; Yang, T. T., et al., 1998), while the interest rate reflects the lognormal process of Black, Derman, and Toy (1990) and the mortality-related risks is assessed by the Lee-Carter model.

However, when the process for the short-term interest rate follows the lognormal process, the closed-form solutions of reverse mortgages are not available. In this article, we use a three-dimensional lattice method which can simultaneously capture the evolution of housing price and short-term interest rate to calculate the fair valuation of reverse mortgages. For the housing price process, we use the Cox-Ross-Rubinstein (CRR, 1979) model to generate the possible states of future housing price. For the process of short-term interest rates, we use the Black-Derman-Toy (BDT, 1990) model to generate the possible states of future spot rates.

Securitization is a financial innovation that emerged in the 1970's in the US financial market. According to Cummins (2004), securitization involves the isolation of a pool of assets or rights to a set of cash flows and the repackaging of the assets or cash flows into securities that are traded in capital markets. The idea of securitizing mortality and/or longevity risks is introduced (see Blake and Burrows (2001) and Blake (2003)). There is an increased interest to model these types of mortality-based securities; hence, the ideas of mortality bonds and mortality swaps are proposed in the literature (Lin and Cox, 2005) and effectively put into practice. Following a similar approach used by Lin and Cox (2005), Wang, Valdez and Piggott (2007) propose a securitization method to hedge the longevity risk by using survivor bonds and survivor swaps for reverse mortgage products.

Following a similar approach used by Lin and Cox (2005), Denuit et al. (2007) and Wang, Valdez and Piggott (2007), this paper proposes a securitization method－crossover bonds－to hedge the crossover risk inherent in reverse mortgage products. For a mortgage reverse insurer, the premium structure of reverse mortgage insurance is determined by setting the present value of total expected claim losses equal to the present value of the premium charges. However, when the actual loss is higher than the expected loss, the insurer will incur an unexpected loss. To offset the potential loss, we design a crossover bond to transfer the crossover risk into the bond investors. The payoff structure of crossover bonds is related to the actual losses and expected losses of reverse mortgages. At each payment date, if the actual loss of reverse mortgage is less than the expected loss, the bond investors can obtain a higher level of coupon rate; otherwise, they can only receive a lower level of coupon rate. Therefore, through the crossover bonds, the reverse mortgage insurers can transfer the unexpected loss into the bondholders. In this article, using the three-dimensional lattice method, we will numerically calculate the fair coupon rates of crossover bonds.

2. Pricing Model of Reverse Mortgage Insurance Contracts

In this section, we first describe the contract structure of reverse mortgages, which provides the basis for our valuation. We then model the dynamics of the spot interest rates, the house prices, and the mortality rates sequentially.

2.1. Reverse mortgage contracts

In U.S. HECM program, borrowers are required to pay 2% of housing values as an upfront mortgage insurance premium (UP_0) and a monthly mortgage insurance premium (MIP_t) according to the annual rate of 0.5% of the outstanding loan balances. Using this predetermined insurance premium structure, we evaluate the present values of expected claim losses and that of insurance premiums, determining the maximum levels of constant monthly payments under the condition satisfying the present values of expected claim losses are equal to that of insurance premiums.

We investigate reverse mortgage with a lump sum payment, analogous to the U.S. HECM program (Szymanoski, 1994). The initial property value, denoted H_0, enables us to determine the lump sum payment. We assume that the loan becomes due and payable only at the borrower's death. The borrower receives a lump sum payment, BAL_0, and does nothing else, because the house is his or her principal residence. BAL_t, the outstanding balance at time *t*, is determined by the outstanding balance at time *t*-1 plus the premium charge with interest accrued. BAL_t can be calculated as follows:

$$BAL_t = \left(BAL_{t-1} + MIP_t\right)\left(1+y\right) \tag{1}$$

where BAL_t $\equiv$ The outstanding loan balance at time *t*

$$BAL_0 = M \cdot H + UP_0 = M \cdot H + 0.02H = (M+0.02)H$$

M $\equiv$ Maximum Level of Mortgages

UP_0 $\equiv$ Upfront mortgage insurance premium at inception

MIP_t $\equiv$ Yearly mortgage insurance premiums at time *t*;

$$MIP_t = 0.005 BAL_{t-1}$$

y $\equiv$ Mortgage interest rate at time *t*

We can recursively obtain the expressions for MIP_t and BAL_t as follows:

$$BAL_t = 1.05^t (M+0.02)(1+y)^t H_0, \tag{2}$$

$$MIP_t = 0.005 BAL_{t-1} = 0.005 \times 1.05^{t-1}(M+0.02)(1+y)^{t-1} H_0. \tag{3}$$

The reverse mortgage contracts involve a range of risks from the insurer's perspective. The outstanding balance usually accumulates at a faster rate than the appreciating rate of the housing value; therefore, if the outstanding balance exceeds the housing value before the loan is settled, the lender starts to incur a loss—namely crossover risk, one of the crucial risks to manage in reverse mortgages. Apparently, this crossover risk is induced by three risk factors: interest rates, house prices and mortality. In the following section, we describe their dynamics, respectively.

2.2. Interest rate process

Generally speaking, under the ordinary economic environment, we can assume the short-term risk-free interest rate is constant. However, in some circumstances, the short-term interest rate changes dramatically. For example, when the central bank suddenly changes monetary policies or oil shocks occur, the short-term interest rate will fluctuate over time. Thus, instead of using constant interest rate, we must assume the short term risk-free interest rate to be stochastic.

There are a number of models of the local process for the short-term interest rate—a normal process (Vasicek, 1977; Jamshidian, 1989), a lognormal process (Dothan, 1978; Black, Derman, and Toy (1990); Black and Karasinski 1991) and a square-root process (Cox, Ingersoll, and Ross (1985)) or others. Among them, lognormal models keep the rate away from zero entirely, while the normal process may fall below zero and some square-root models make zero into a "reflecting barrier."

In this article, we use the lognormal model to describe the evolution of short-term interest rates as follows:

$$d\ln r_t = \left(\theta_t + \frac{\partial \ln \sigma_t / \partial t}{\sigma_t} \ln r_t\right) dt + \sigma_t^r \cdot dW_t \,, \tag{4}$$

where r_t is the instantaneous spot rate at time *t;* θ_t is long-term interest rate parameter; σ_t^r is the volatility vector of the spot rate at time *t* and satsifies

$\sigma_t^r = \sigma_r [1,0]'$, that is, $|\sigma_t^r| = \sigma_r$; $|\cdot|$ denotes the Euclidean norm in R^2; and W_t represents a 2-dimensional standard Brownian motion under a Risk-neutral probability measure Q and satisfies $W(t) = [W_r(t), W_H(t)]'$.

Note that Equation (4) is a continuous time limit of the Black-Derman-Toy one-factor model (BDT model), incorporating two independent functions of time, θ(t) and σ(t), chosen so that the model fits the term structure of spot interest rates and the term structure of spot rate volatilities. Assuming a different lognormal short rate distribution for each future time allows both mean and variance to depend on time. In contrast to the Vasicek model, in the lognormal representation the short rates are log-normally distributed; with the resulting advantage that interest rates cannot become negative.

2.3. House price model

For mortgage valuation, it typically assumes that housing price follows a stochastic Geometric Brownian motion (GBM) process (Cunningham and Hendershott, 1984; Kau, J. B., et al., 1992; Chinloy and Megbolugbe, 1994; Hilliard, J. E., et al., 1998; Szymanoski, 1994; Yang, T. T., et al., 1998). This process is also known as a continuous time limit of random walk with drift for the dynamics of instantaneous rate of returns. Therefore, under the risk-neutral measure Q, we assume that the house price process is governed by

$$\frac{dH_t}{H_t} = (r_t - \delta_t)\,dt + \sigma_t^H \cdot dW_t, \tag{5}$$

where δ_t is a maintenance yield (or rental rate) for the house; σ_t^H is the volatility vector of the housing price and satsifies $\sigma_t^H = \sigma_H \left[\rho_{Hr}, \sqrt{1-\rho_{Hr}^2} \right]'$, that is, $\left|\sigma_t^H\right| = \sigma_H$; and ρ_{Hr} is the correlation coefficient between the interest rate process and the house price process.

2.4. Mortality model

Ever since Lee and Carter presented their original work in 1992, the Lee-Carter model has been widely used in mortality trend fitting and projection. The Census Bureau population forecast has used it as a benchmark for the long-run forecast of U.S. life expectancy. The two most recent Social Security Technical Advisory Panels have suggested the Trustees to adopt this method or other methods consistent with it (Lee and Miller, 2001).

In this article, we use the Lee-Carter model to assess the mortality-related risks. Let $m_{x,t}$ be the central death rate for age *x* at time *t*. The Lee-Carter model assumes:

$$ln\left(m_{x,t}\right) = \alpha_x + \beta_x k_t + e_{x,t}, \tag{6}$$

where α_x represents the age pattern of death rates, β_x describes the pattern of deviations from the age *x* profile when the parameter *k* varies, k_t explains the change of mortality over time t, and $e_{x,t}$ describes the error term, which is expected to be white noise with zero mean and a relatively small variance (Lee, 2000).

The Lee-Carter model cannot be fitted by the ordinary least square approach, because all variables on the right side of the model are unobservable. Moreover, this model is obviously over-parameterized. We use the singular value decomposition approximation (Lee and Carter, 1992) to fit the solutions of the parameters. To obtain a unique solution, a normalization conditions is imposed such that the β_x terms sum to unity and the k_t terms sum to zero, i.e.,

$$\sum_t k_t = 0 \text{ and } \sum_x \beta_x = 1. \tag{7}$$

Then α_x becomes the average value of $ln\left(m_{x,t}\right)$. For each age group *x*, we can obtain $\hat{\beta}_x$ by regressing $\log\left(m_{xt}\right) - \hat{\alpha}_x$ on $\hat{k}_t$ without a constant term.

Following Lee and Carter (1992), we forecast future values of k_t with

$$k_t = k_{t-1} + z + \varepsilon_t, \tag{8}$$

where *z* is the drift parameter, and ε_t is a sequence of independent and identically normal distributions with mean 0 and variance σ^2. We assume that the values $k_1, \ldots,$ k_{t_0} are known but that k_{t_j} are unknown and must be forecast, where $t_j = t_0 + j$, for any natural number *j*. By virtue of Equation (8), we have

$$k_{t_j} = k_{t_0} + jz + \sum_{i=1}^{j} \varepsilon_{t_i}, \tag{9}$$

Moreover, conditional on t_0, k_{t_j} is normal distributed with mean $k_{t_0} + jz$ and variance $j\sigma^2$.

Let $p_{x_0}(t_0)$ denote the one-year survival probability that an x_0-aged person in calendar year t_0 reaches age x_0 + 1. We assume that the age-specific mortality rates are constant within bands of age and time but may vary from one band to the next. Specifically, given any integer age x_0 and calendar year t_0, we suppose that

$$m_{x_0+\xi,\, t_0+\tau} = m_{x_0, t_0} \text{ for } 0 \le \xi, \tau < 1. \tag{10}$$

Thus, the one-year survival probability can be calculated as $p_{x_0}(t_0)=\exp(-m_{x_0,t_0})$.

Let ${}_{t_n}p_{x_0,t_0}$ denote the n-year survival probability that an x_0-aged person in calendar year t_0 reaches age x_0+n, which is

$$ {}_{t_n}p_{x_0,t_0} = \exp\left(-\sum_{j=0}^{n-1} m_{x_j,t_0+j}\right) = \exp\left(-\sum_{j=0}^{n-1}\exp\left(\alpha_{x_0+j}+\beta_{x_0+j}k_{t_0+j}\right)\right). \qquad (11) $$

2.5. Pricing model for reverse mortgage insurance

We determine the lump sum payment BAL_0 when the present value of the insurance premiums covers the present value of expected losses from future claims. Through the pricing process, it is more convenient to set the valuation date t_0 to 0. Thus, at the valuation date t_0 (=0), the money market account is defined by

$$ B(t) = \exp\left(\int_0^t r_u du\right). \qquad (12) $$

We also assume that the market is complete and without arbitrage. Based on the arbitrage pricing theory, the value of the reverse mortgage insurance, which is also the present value of the expected losses from future claims, equals the expectation of discounted future cash flows under risk-neutral measure *Q*. Let x_0 be the age of the borrower at time t_0; then, the value of the reverse mortgage insurance is of the form:

$$ PVMIP = UP_0 + \sum_{j=1}^{N} E_Q\left[{}_{t_j}p_{x_0,t_0}\frac{MIP_{t_j}}{B(t_j)}\right] $$

$$ = \sum_{j=1}^{N} E_Q\left[\left({}_{t_{j-1}}p_{x_0,t_0} - {}_{t_j}p_{x_0,t_0}\right)\frac{Max\left(BAL_{t_j}-H_{t_j},0\right)}{B(t_j)}\right] = PVEL, \qquad (13) $$

where

$PVMIP$	$\equiv$	Present value of total mortgage insurance premiums at inception (time t_0)
$PVEL$	$\equiv$	Present value of total claim losses at inception
N	$\equiv$	The number of years that borrowers with age *x* will live until they reach 110 years of age
${}_{t_j}p_{x_0,t_0}$	$\equiv$	The probability that a borrower of age *x* at inception will survive at age *x+j*

Under the assumption that mortality rate and financial risk are independent, we can rewrite Equation (13) as follows:

$$PVMIP = UP_0 + 0.005(M + 0.02)H_0 \sum_{j=1}^{N} 1.05^{j\text{-}1}(1+y)^{j\text{-}1} p(t_0, t_j) S_{x_0}\left(t_j\right), \tag{14}$$

$$PVEL = \sum_{j=1}^{N} \left[S_{x_0}\left(t_{j-1}\right) - S_{x_0}\left(t_j\right) \right] C\left(t_j\right), \tag{15}$$

where $p(t,T)$ denote the price of a zero-coupon bond issued at time *t* that pays \$1 at time *T*, $t \le T$; $S_{x_0}\left(t_n\right) = E_Q\left[{}_{t_n} p_{x_0,t_0} \right]$; and $C\left(t_j\right)$ is of the form:

$$C\left(t_j\right) = E_Q\left[\frac{Max\left(BAL_{t_j} - H_{t_j}, 0\right)}{B(t_j)} \right]. \tag{16}$$

However, when the process for the short-term interest rate follows the lognormal process defined in Equation (4), the closed-form solutions of $C\left(t_j\right)$ are not available. To numerically obtain the values of $C\left(t_j\right)$, there are many techniques in numerical method of option pricing, such as finite difference methods, lattice or tree methods, and Monte Carlo methods.

In this article, we use a three-dimensional lattice method which can simultaneously capture the evolution of housing price and short-term interest rate. For the housing price process, we use the Cox-Ross-Rubinstein (CRR) model to generate the possible states of future housing price. This means that, if $H_0{=}H$ is the asset value at time 0, then, after one period, at time 1, it can rise to *uH* or decrease to *dH*, where *u* and *d* represent, respectively, the magnitude of one up step and the magnitude of one down step. Consequently, the underlying asset price evolution can be represented by a binomial tree where each node corresponds to one possible value of the asset price. Note that, in the CRR model, since the log-change of housing price is modeled by a random walk with drift, then by taking the limit of the number of periods equal to infinity, the continuous-time limit of random walk with drift model for log-change of housing price will become the GBM process defined in Equation (5).

[Insert Figure 1]

For the process of short-term interest rates, we use the Black-Derman-Toy model to generate the possible states of future spot rates. The BDT model is a one-factor short-rate (no-arbitrage) model — all security prices and rates depend only on a single factor, the short rate — the annualized one-period interest rate. The current structure of long rates (yields on zero-coupon Treasury bonds) for various maturities and their estimated volatilities are used to construct a tree of possible future short rates (for details, please see Black, Derman and Toy (1990)). This tree can be used to value interest-rate-sensitive securities such as reverse mortgage contracts.

Similarly, when the number of periods approaches to infinity, the process of short-term interest rates governed by BDT model become a lognormal process defined in Equation (4). Combining the CRR model and BDT model, Figure 1 depicts the three-period lattice model to numerically obtain the values of $C\left(t_j\right)$. The initial advance BAL_0 can be determined by setting the present value of total expected claim losses equal to the present value of the premium charges, namely $PVMIP = PVEL$.

3. Securitization for Reverse Mortgage Insurance

Suppose the lender holds a portfolio of L loans. At time 0, all the borrowers are of the different ages ranging from aged 62 to aged 100. Each borrow a lump sum against their home property. When the outstanding balance exceeds the housing value before the loan is settled, the insurer starts to incur a loss. Therefore, for a mortgage reverse insurer, the premium structure of reverse mortgage insurance is determined by setting the present value of total expected claim losses equal to the present value of the premium charges. However, when the actual loss is higher than the expected loss, the insurer will incur an unexpected loss. To offset the potential loss, we design a principal-guaranteed crossover bond to transfer the unexpected losses induced by crossover risk into the bond investors.

The payoff structure of the crossover bond is related to the actual loss and the expected loss. Similar to Treasury bonds, the crossover bonds pay interest at each coupon payment date and the principal at maturity. Unlike the Treasury bonds, when the actual loss is less than the expected loss at the coupon payment date, the bond investors will receive a higher level of coupon rate than that of the Treasury bonds with the same maturity. otherwise, the bond investors will receive a lower level of coupon rate. Since the closed form solution of the crossover bonds are hard to derive, using the three-dimensional lattice method, we can calculate the fair valuation of the crossover bonds.

4. Numerical analysis

For numerically analyze the impacts of longevity risk, interest rate risk and housing pricing risk on the pricing reverse mortgage, we first describe the parameters for the dynamics of interest rate, housing price and mortality rate, then we present the numerical results for the loan-to-value (LTV) ratios as well as their sensitivity analysis. Finally, we compute the fair coupon rates of the crossover bonds with maturity up to thirty years.

First, as shown in Figure 2, we employ the Treasury zero rates from GreTai Securities Market in Taiwan to calibrate the parameters of BDT model. Using the zero rate curve, the minimum, medium and maximum of the spot rate tree are depicted in Figure 2. The higher the spot rate volatility, the larger the difference between the minimum and maximum of the future spot rates. Without loss of generality, we apply the mediums at each time period of the BDT model as the corresponding mortgage rates of reverse mortgage. As shown in Figure 3, the curves of mortgage rates for different spot rate volatilities are very close to each other.

[Insert Figure 2]

[Insert Figure 3]

We employ Taiwan mortality data from the Department of Statistics, Ministry of the Interior in Taiwan to calibrate the parameters of the Lee-Carter model. Final age all lives are assumed to end is 110. The pattern of empirical mortality rates appear in Figure 4. Applying the Lee-Carter model, we depict the survival probabilities $S_{x_0}\left(t_n\right)$ for $x_0 = 65$, 70, 75 and 80 in Figure 5. The higher is the age—ceteris paribus—the lower is the survival probability.

[Insert Figure 4]

[Insert Figure 5]

Using the three-dimensional lattice method, we first present the numerical results for a representative base case. For the parameters of the base case, the initial housing value is assumed to be $5,000,000, i.e., H_0=5,000,000; the housing price volatility is 45%; and the interest rate volatility is 2%. Figure 6 depicts the LTV ratios for different ages. The lower is the age—ceteris paribus—the lower is the LTV ratio. For its economic implication, the present value of the house is the sum of the present value of future rental incomes. According to the reverse mortgage mechanism, the borrower uses the rental income after his or her death in exchange for the lump sum payment at the inception. An older borrower can borrow more money since his or her expected death comes sooner and the present value of the rental income after death is greater.

[Insert Figure 6]

We next examine the sensitivity of the LTV ratios by varying the level of housing price volatility and interest rate volatility in Table. From Table 1, it can be seen that the higher the interest rate volatility, the lower the LTV ratio. For its economic implication, the present value of the house is the sum of the present value of future rental incomes. The higher interest rate volatility may lead to a higher level of interest rate and hence results in a lower present value of the house as well as a lower LTV ratio. Similarly, since the higher housing price volatility may contribute to a higher level of housing price, the higher is the housing price volatility, the higher is the LTV ratio. Note that, form Table 1, the impact of housing price volatility is more significant than that of interest rate volatility on LTV ratio, which indicates that for pricing reverse mortgage it is crucial to estimate the volatility of housing price precisely.

[Insert Table 1]

For the securitization of reverse mortgage, we assume the payoff structure of the principal-guaranteed crossover bond is related to the actual losses and the expected losses. The actual loss at each period is calculated according to the distribution of borrower age and gender as follows:

$$AL_t = \sum_{g=f \text{ or } m} \sum_{x=62}^{100} w_g w_x \times AL_t^{g,x} , \tag{16}$$

where AL_t is the actual loss at time *t*; w_g is the gender weight; w_{age} is the age weight; and $AL_t^{f,x}$ ($AL_t^{m,x}$) is the actual loss for a female (male) with age *x* and satisfies:

$$AL_{t_j}^{g,x_0} = {}_{t_{j-1}}\left|q_{x_0}^g\right. \frac{Max\left(BAL_{t_j}^{g,x_0} - H_{t_j}, 0\right)}{B(t_j)}, \tag{17}$$

where ${}_{t_{j-1}}\left|q_{x_0}^g\right. = {}_{t_{j-1}}p_{x_0,t_0}^g - {}_{t_j}p_{x_0,t_0}^g$ denotes an individual live for t_{j-1} years and will die in one year. Using the three-dimensional lattice method, we can calculate the actual losses, together the expected losses, for different ages at each period. For example, the expected losses for the next thirty years for age *x*=65, 70, 75 and 80 are depicted in Figure 7. Since the expected loss at time *t* is determined by the probability ${}_{t-1}\left|q_x\right.$ and the relation between ${}_{t-1}\left|q_x\right.$ and time period in Figure 8 also exhibits a humped curve, it is reasonable that the relationship between the expected losses and time period exhibits a humped curve.

[Insert Figure 7]

[Insert Figure 8]

The payoff structure of the crossover bond is defined as follows. Similar to the Treasury bonds, the bond investors will receive the interest at each coupon payment date and the principal at maturity. However, unlike the Treasury bonds, when the actual loss is larger than the expected loss at the coupon payment date, the bond investors will receive coupon rate equal to 0.5%. Otherwise, the bond investors will receive a higher level of coupon rate than that of the Treasury bonds with the same maturity. In addition, according to the gender and age distribution of HECM loan borrowers (Bishop and Shan, 2008), the female weight is assumed to be 0.6 and the age weight w_x are given in Figure 9. Using the three-dimensional lattice method, in Table 2 we calculate the fair coupon rate of the principal-guaranteed crossover bond with different time to maturities. As the time to maturity increases, both the coupon rate and the markup increase. Therefore, when the actual loss is larger than the expected loss at the coupon payment date, the issuer of crossover bond can partly hedge the unexpected loss and the bond holders also receive a minimum coupon rate and principal at maturity. Otherwise, the crossover bond holders can receive a higher coupon than the Treasury bond with the same time to maturity. The win-win situation probably makes the bond attractive and hence the issuers can successfully transfer the crossover risk to the bond market.

5. Conclusion

In addition to government-sponsored social security systems and employer-sponsored retirement plans, reverse mortgages are becoming remarkably popular in the last few years since it allows retirees to convert their substantial home equities into either a lump sum or annuity income and to remain in their homes until they die, sell or vacate their homes to live elsewhere. From the insurer's perspective, the reverse mortgages involve a range of risks, including housing price risk, interest rate risk and longevity risk. In this paper, under the consideration of housing price risk, interest rate risk and longevity risk, we provide a three-dimensional lattice method which simultaneously captures the evolution of housing price and short-term interest rate and numerically calculate the fair valuation of reverse mortgages.

For a mortgage reverse insurer, the premium structure of reverse mortgage insurance is determined by setting the present value of total expected claim losses equal to the present value of the premium charges. However, when the actual loss is higher than the expected loss, the insurer will incur an unexpected loss. To partly hedge the unexpected loss, we design a principal-guaranteed crossover bond, the payoff structure of which is related to the actual losses and expected losses. Therefore, through the crossover bonds, the reverse mortgage insurers can partly transfer the crossover risk into the bond investors.

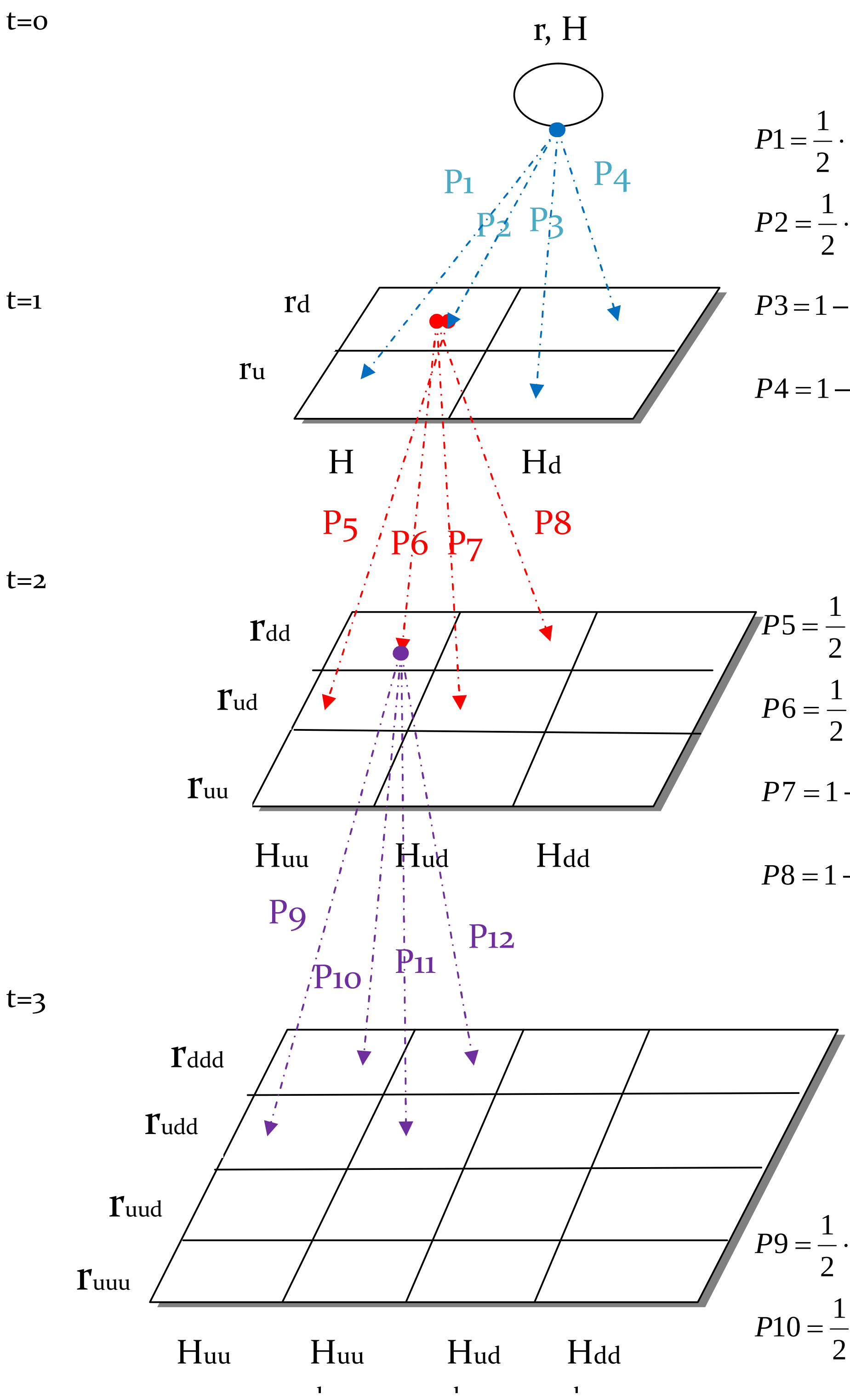
For further researches, since the work of Clark (1973), it has been recognized that the dynamics of asset returns present some commonly observed statistical properties, known as *stylized empirical facts* in the financial econometrics (Cont, 2001). Therefore, the dynamics of housing returns may not be adequately described by geometric Brownian motion with constant drift and volatility. From the numerical analysis, we find that the impact of housing price volatility is more important than that of interest rate volatility on pricing reverse mortgage; therefore, it is an interesting topic to incorporate the *stylized facts* with the housing price process such as exponential Lèvy processes or GARCH Lèvy models for pricing reverse mortgages.

Table 1 Loan-to-Value Ratios

Volatility		Gender	Age			
Interest Rate	Housing Price		65	70	75	80
vr=0.01	vs=0.40	Female	0.3217	0.3853	0.4451	0.4908
		Male	0.3347	0.3935	0.4462	0.4829
vr=0.02	vs=0.40	Female	0.3216	0.3852	0.4450	0.4907
		Male	0.3347	0.3934	0.4461	0.4826
vr=0.03	vs=0.40	Female	0.3215	0.3851	0.4449	0.4904
		Male	0.3345	0.3932	0.4458	0.4822
Volatility		Gender	Age			
Interest Rate	Housing Price		65	70	75	80
vr=0.01	vs=0.40	Female	0.4706	0.5515	0.6228	0.6714
		Male	0.4847	0.5599	0.6240	0.6634
vr=0.02	vs=0.45	Female	0.4705	0.5514	0.6227	0.6712
		Male	0.4846	0.5598	0.6239	0.6631
vr=0.03	vs=0.45	Female	0.4703	0.5512	0.6225	0.6708
		Male	0.4844	0.5596	0.6236	0.6626
Volatility		Gender	Age			
Interest Rate	Housing Price		65	70	75	80
vr=0.01	vs=0.50	Female	0.7447	0.8423	0.9029	0.9195
		Male	0.7556	0.8427	0.8922	0.9033
vr=0.02	vs=0.50	Female	0.7446	0.8422	0.9028	0.9191
		Male	0.7555	0.8425	0.8920	0.9028
vr=0.03	vs=0.50	Female	0.7443	0.8420	0.9024	0.9185
		Male	0.7551	0.8422	0.8915	0.9021

Table 2 Fair Coupon Rates of Crossover Bonds

Time to Maturity	Coupon Rate		The Corresponding Zero Rate	Markup
	Actual Loss >Expected Loss	Actual Loss ≤Expected Loss		
10	0.50%	2.10%	1.66%	+44bps
20	0.50%	2.62%	1.88%	+74bps
30	0.50%	3.09%	2.00%	+109bps



$$P1 = \frac{1}{2} \cdot \frac{e^{r\Delta t} - d}{u - d}$$

$$P2 = \frac{1}{2} \cdot \frac{e^{r\Delta t} - d}{u - d}$$

$$P3 = 1 - \frac{1}{2} \cdot \frac{e^{r\Delta t} - d}{u - d}$$

$$P4 = 1 - \frac{1}{2} \cdot \frac{e^{r\Delta t} - d}{u - d}$$

$$P5 = \frac{1}{2} \cdot \frac{e^{r_d \Delta t} - d}{u - d}$$

$$P6 = \frac{1}{2} \cdot \frac{e^{r_d \Delta t} - d}{u - d}$$

$$P7 = 1 - \frac{1}{2} \cdot \frac{e^{r_d \Delta t} - d}{u - d}$$

$$P8 = 1 - \frac{1}{2} \cdot \frac{e^{r_d \Delta t} - d}{u - d}$$

$$P9 = \frac{1}{2} \cdot \frac{e^{r_{dd} \Delta t} - d}{u - d}$$

$$P10 = \frac{1}{2} \cdot \frac{e^{r_{dd} \Delta t} - d}{u - d}$$

$$P11 = 1 - \frac{1}{2} \cdot \frac{e^{r_{dd} \Delta t} - d}{u - d}$$

$$P12 = 1 - \frac{1}{2} \cdot \frac{e^{r_{dd} \Delta t} - d}{u - d}$$

Figure 1 The Three-Period Lattice method for Pricing Reverse Mortgage Insurance Contracts

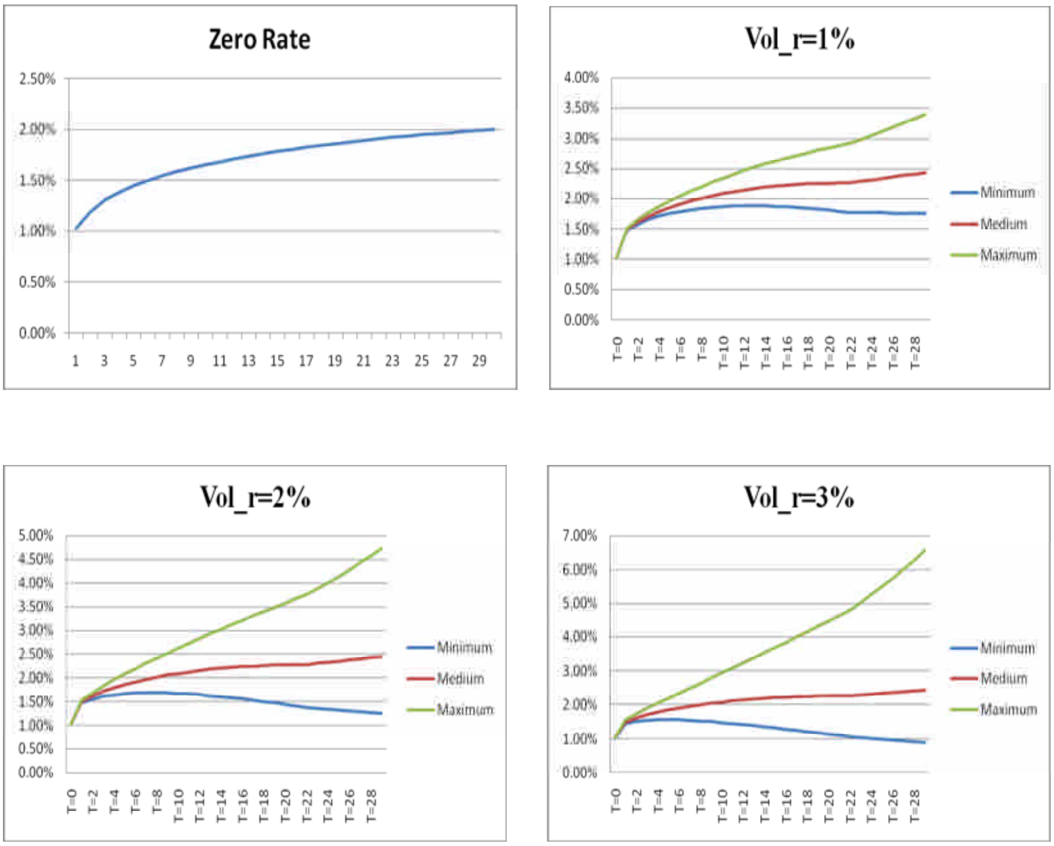


Figure 2 Zero Rate Curve and the Range of BDT model

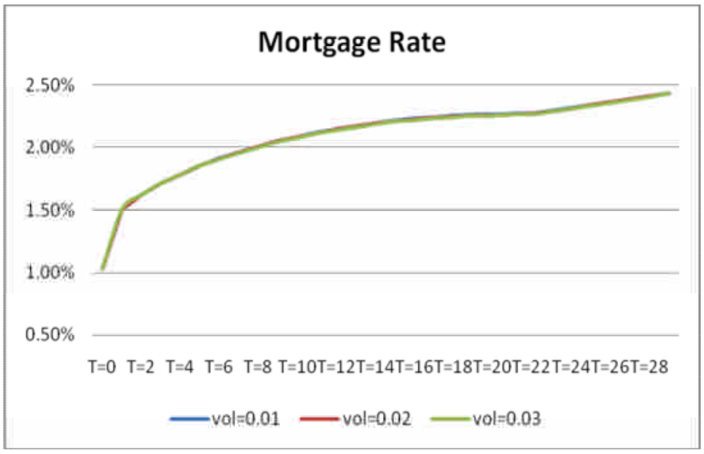
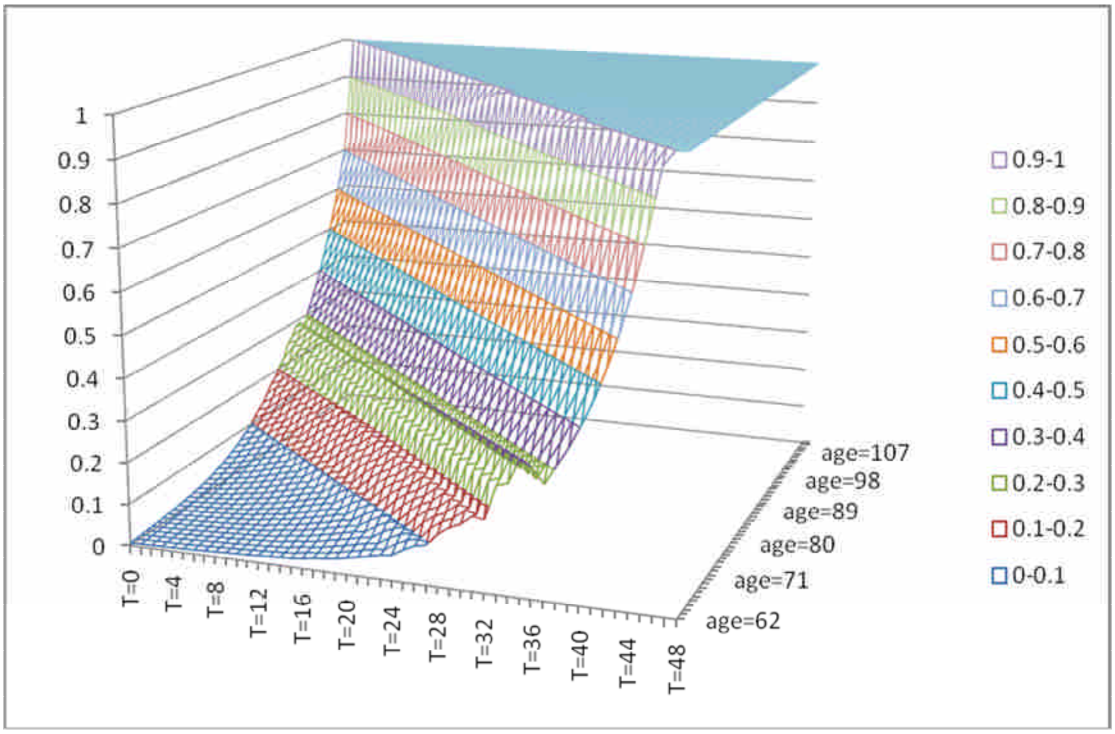
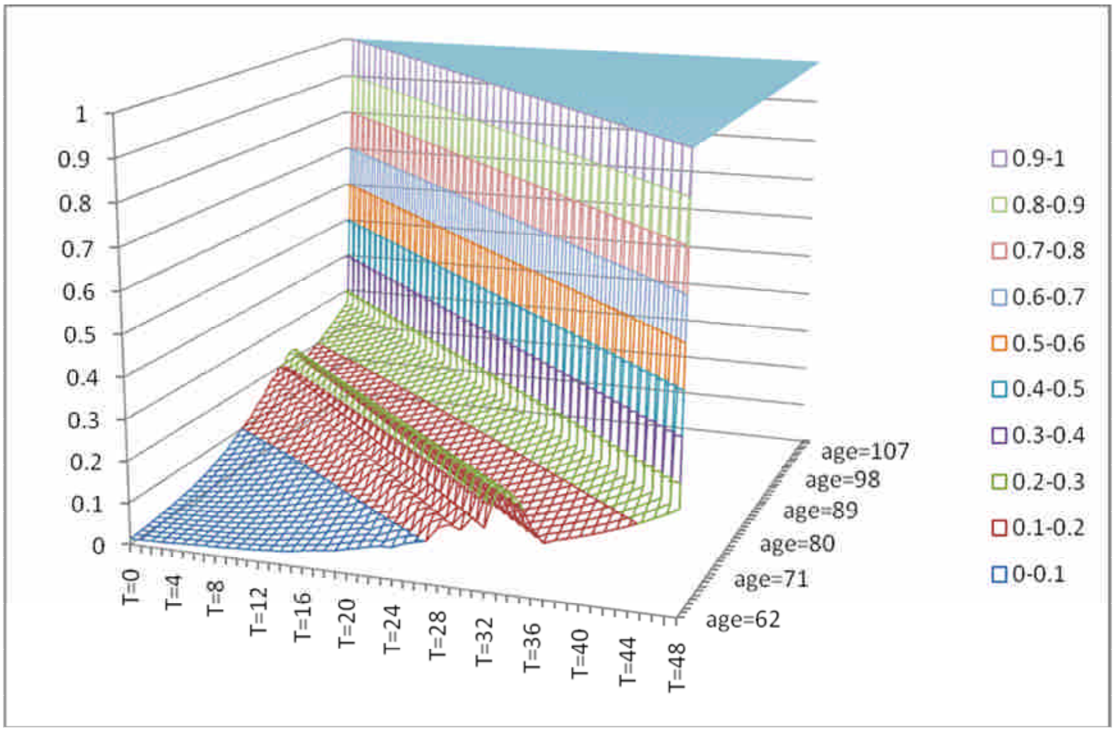


Figure 3 Mortgage Rates for different Spot Rate Volatilities



Panel A Female Mortality Rates



Panel B Male Mortality Rates

Figure 4 Taiwan Mortality Rates for Different Ages

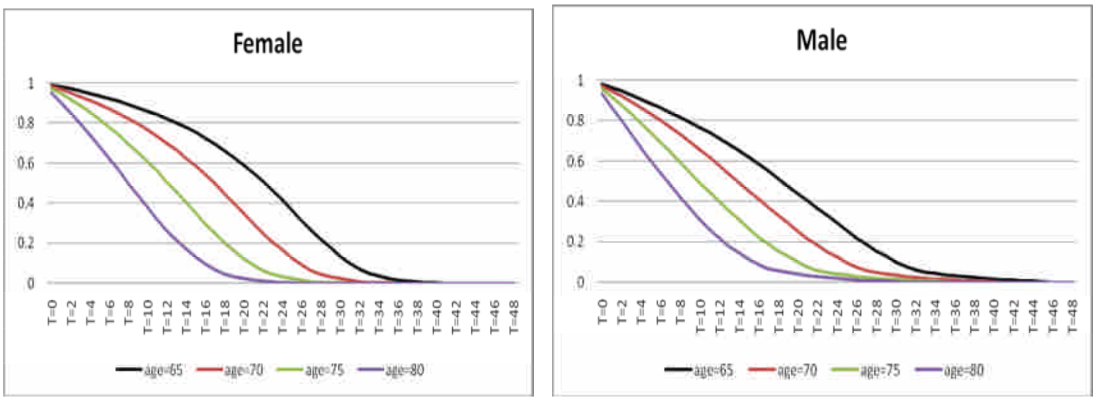


Figure 5 Survival Probabilities for Different Ages

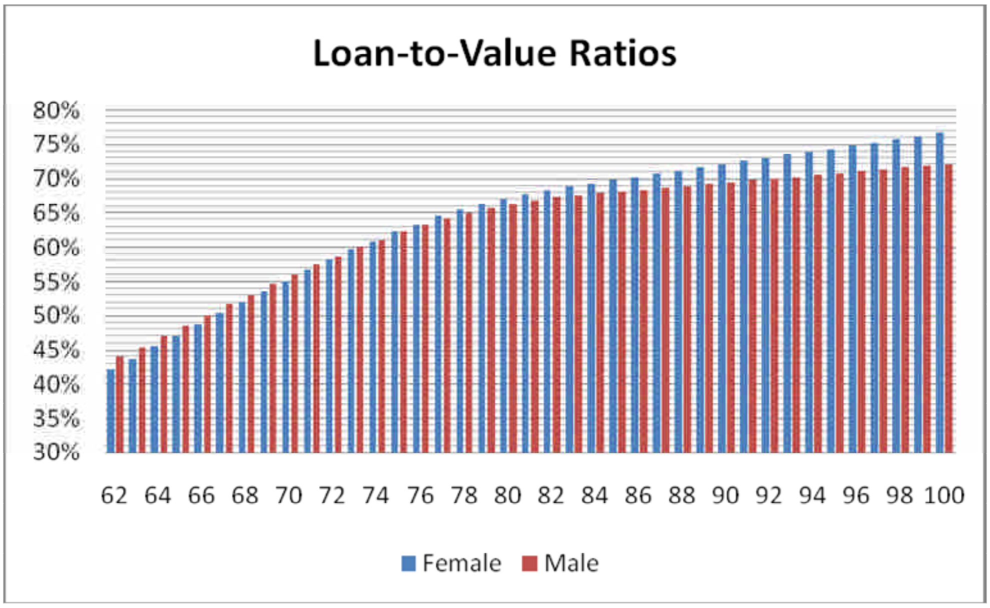


Figure 6 Loan-to-Value Ratios for Different Ages with $\sigma_H = 45\%$ and $\sigma_r = 2\%$

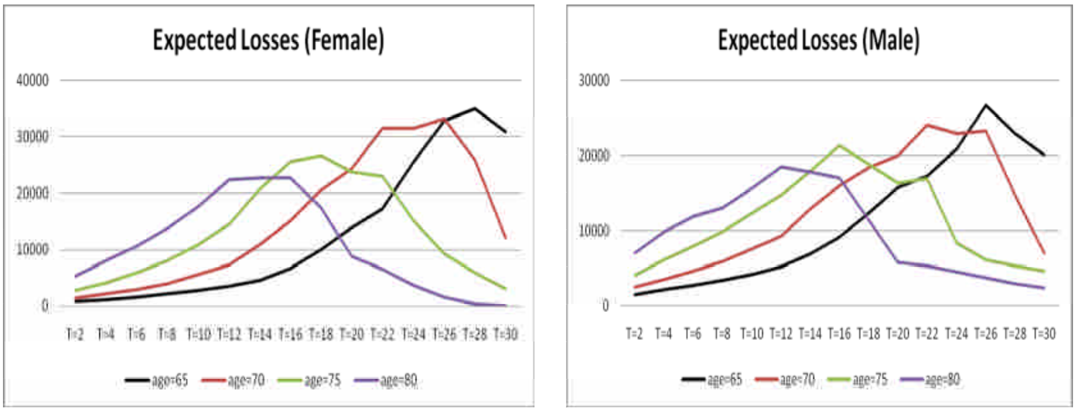


Figure 7 Expected Losses for Different Ages

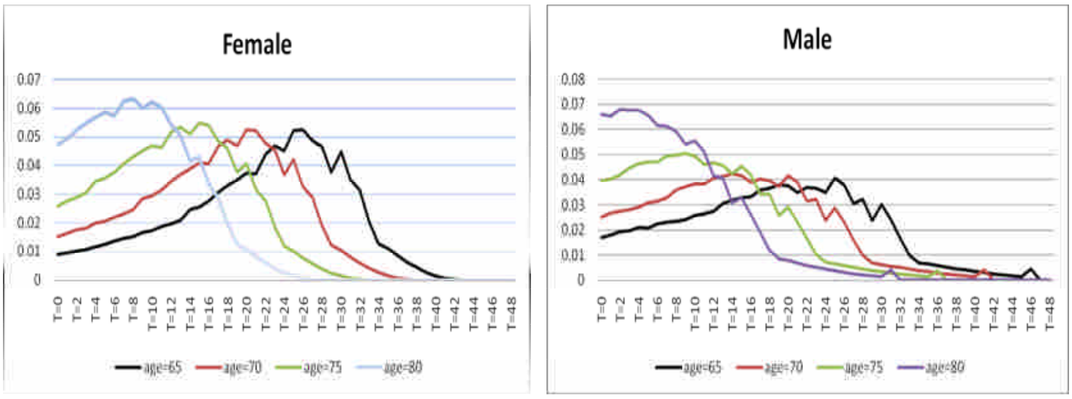


Figure 8 The Relationship Between ${}_{t-1}|q_x$ and Time Period

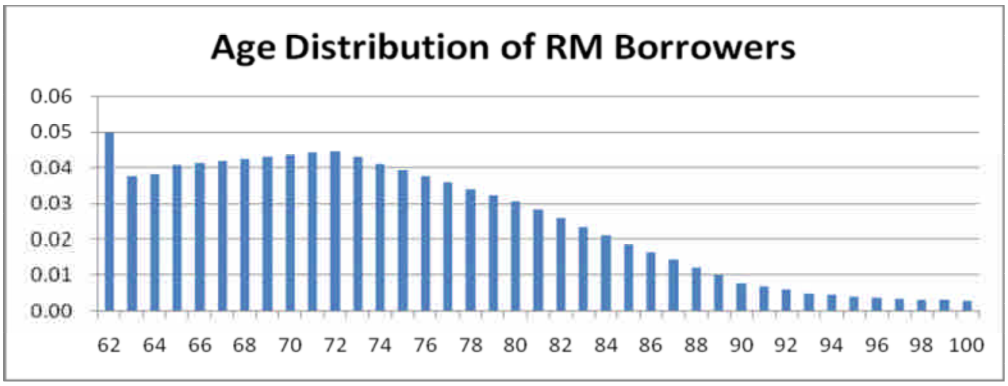


Figure 9 Age weights for the Crossover Bond

Reference

Blake, D., 2003. Reply to survivor bonds: a comment on Blake and Burrows. Journal of Risk and Insurance, 70, 2, 349-351.

Blake, D., Burrows, W., 2001. Survivor bonds: helping to hedge mortality risk. Journal of Risk and Insurance, 68, 2, 339-348.

Black, F., Derman, E., Toy, W., 1990. A one-factor model of interest rates and its application to treasury bond options. Financial Analysts Journal, 46, 1, 33-39.

Boehm, T. P., Ehrhardt, M. C., 1994. Reverse mortgages and interest rate risk. Journal of the American Real Estate and Urban Economics Association, 22, 2, 387-408.

Black, F., Karasinski P., 1991. Bond and option pricing when short-term rates are lognormal. Financial Analysts Journal, 47, 4, 52-59.

Chinloy P., Megbolugbe, I. F., 1994. Reverse Mortgage: Contracting and Crossover Risk. Journal of American Real Estate and Urban Economics Association, 22, 2, 367-386.

Clark, P.K., 1973. A subordinated stochastic process model with finite variance for speculative prices. Econometrica 41, 135–156.

Cont, R., 2001. Empirical properties of asset returns: Stylized facts and statistical issues, Quantitative Finance, 1, pp. 1–14.

Cox, J. C., Ingersoll, J., Ross, S. A., 1985. A theory of the term structure of interest rates. Econometrica, 53, 2, 385-407.

Cox, J. C., Ross, S. A., Rubinstein, M., 1979. Option pricing: a simplified approach. Journal of Financial Economics, 7, 229-263.

Cummins, J. D., 2004. Securitization of life insurance assets and liabilities.submitted to TIAA-CREF Institute.

Cunningham, D., Hendershott, P., 1984. Pricing FHA Mortgage Default Insurance. Housing Finance Review, 3, 4, 373-392.

Denuit, M., Devolder, P., Goderniaux, A.C., 2007. Securitization of longevity risk: pricing survivor bonds with wang transform in the Lee-carter framework. The Journal of Risk and Insurance, 74, 1, 87-113.

Dothanm, M., 1978. On the Term Structure of Interest Rates. Journal of financial economics, 7, 229-264.

Hilliard, J. E., Kau, J. B., Slawson V. C. Jr., 1998. Valuing prepayment and default in a fixed-rate mortgage: A bivariate binomial options pricing technique. Real Estate Economics, 26, 3, 431-468.

Jamshidian, F., 1989. An exact bond option formula. Journal of Finance, 44, 1, 205-209.

Kau, J. B., Keenan, D., Muller, W., Epperson J., 1992. A generalized evaluation model for fixed-rate residential mortgages. Journal of Money, Credit and Banking, 24, 3, 279-299.

Lee, R., 2000. The Lee-Carter method for forecasting mortality, with various extensions and applications. North American Actuarial Journal, 4, 80-93.

Lee, R. D., Carter, L. R., 1992. Modeling and Forecasting U.S. Mortality. Journal of the American Statistical Association, 87, 659-675.

Lee, R. D., Miller, T., 2001. Evaluating the performance of the Lee-Carter method for forecasting mortality. Demography, 38, 4, 537-549.

Lin, Y., Cox, S. H., 2005. Securitization of mortality risks in life annuities. Journal of Risk and Insurance, 72, 2, 227-252.

Ma, S., Kim, G., and Lew. K., 2007. Estimating Reverse Mortgage Insurer's Risk Using Stochastic Models. Paper presented at the Asia-Pacific Risk and Insurance Association 2007 annual meeting.

Mitchell, O.S., Piggott, J., 2004. Unlocking Housing Equity in Japan. Journal of the Japanese and International Economies, 18, 4, 7-18.

Phillips, W.A., Gwin, S. B., 1992. Reverse Mortgage. Transactions o the Society of Actuaries, 44, 289-323.

Rodda, D. T., Lam, K., and Youn, A., 2004. Stochastic Modeling of Federal Housing Administration Home Equity Conversion Mortgages with Low-Cost Refinancing, Real Estate Economics 32, 4, 589-617.

Bishop, T, B., and Shan, H., 2008. Reverse Mortgages: A Closer Look at HECM Loans.

Taylor, S., 1975. Paul L'evy, Bull. London Math. Soc., 7 , pp. 300–320.

Tse, Y.K. 1995. Modelling Reverse Mortgages. *Asia Pacific Journal of Management*. 12(2): 79-95

Szymanoski, E. Jr., 1994. Risk and the Home Equity Conversion Mortgage. Journal of American Real Estate and Urban Economics Association, 22, 2, 347-366.

Vasicek, O., 1997. An equilibrium characterization of the term structure. Journal of Financial Economics, 5, 177-188.

Wang, L., Valdez, E.A., and Piggott, J. 2007. Securitization of Longevity Risk in Reverse Mortgages. Working paper, available at SSRN: http://ssrn.com/abstract=1087549

Yang, T. T., Buist, H., and Megbolugbe, I. F., 1998. An analysis of the ex ante probabilities of mortgage prepayment and default. Real Estate Economics, 26, 4, 651-676.

Zhai, D.H. 2000. Reverse Mortgage Securitizations: Understanding and Gauging the Risks. Structure Finance. Moody's Investors Service Special Report