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Pricing Life Expectancy

A Framework for Longevity Options

Wolfgang Murmann | Longevity Six in Sydney | September 2010

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Agenda

1. Introduction
2. Options Pricing Framework
   a) The data
   b) Description of the underlying
   c) The forward issue
   d) The model
   e) Sensitivity analysis
3. Conclusion

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Introduction

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Introduction

Master thesis: Pricing Life Expectancy – A Framework for Longevity Options (July 2009)

Prof David Blake
(Supervisor)

- Professor of Pension Economics at Cass Business School
- Director of the Pensions Institute (established in 1996)
- Chairman of Square Mile Consultants, a training and research consultancy
- Co-Founder with JPMorgan and Watson Wyatt of the LifeMetrics Indices
- David Blake was a student at the London School of Economics in the 1970s and early 1980s, gaining his PhD on UK pension fund investment behaviour in 1986

Dr Dirk Nitzsche
(Supervisor)

- Senior lecturer at Cass Business School (Faculty of Finance)
- Areas of Interest: Applied Econometrics, Asset Pricing, Financial Markets, Fund Management, Financial Economics, Pension Funds
- Course directorships: MSc Quantitative Finance, MSc Financial Mathematics
- Before joining Cass he spent 6 years at the Business School at Imperial College
- Gained his PhD at the University of Newcastle

Wolfgang Murmann

- More than 10 years of professional experience in banking / investment banking
- Former roles (amongst others): Interest rates derivatives structuring, Exotic equity derivatives structuring
- Current role: Alternative Asset Solutions; main task: Develop and market capital markets solutions for longevity risk
- BBA in Banking, Finance and Management at Frankfurt School of Finance & Management (2006)
- MSc in Finance at Cass Business School (2009)

Options Pricing Framework
The data

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The data

- **Data provider:** *Statistisches Bundesamt* (Federal Statistical Office), Wiesbaden (Germany)
- **Data input:** Mortality tables for West Germany (1957 – 2005)
- **Age groups:** 0 - 89 years
- **Note:** Mortality data for World War I (1914 – 1918) and II (1939 – 1945) are not available

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Options Pricing Framework

Description of the underlying

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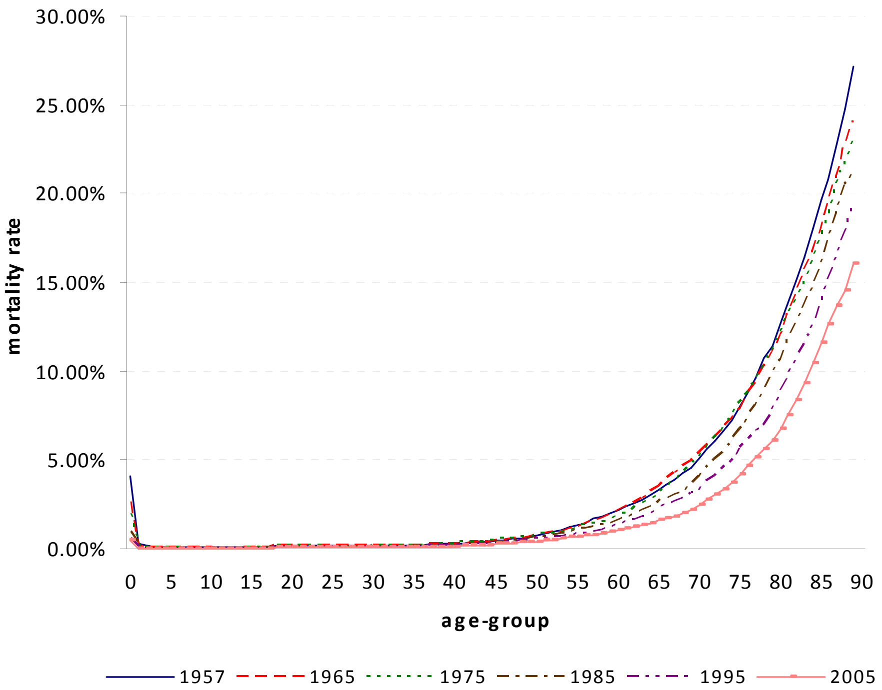
Description of the underlying

Mortality distribution

Mortality distributions for German males (1957 – 2005)
Percentage



Source: Statistisches Bundesamt

Main findings

- Fat left tail (high infant mortality)
- Relatively flat core
- Exponentially increasing mortality rates for older ages
- **Mortality improvements over time**
  → Decreasing mortality rates

Description of the underlying

Excursus: Fitting the spot distribution

Weibull

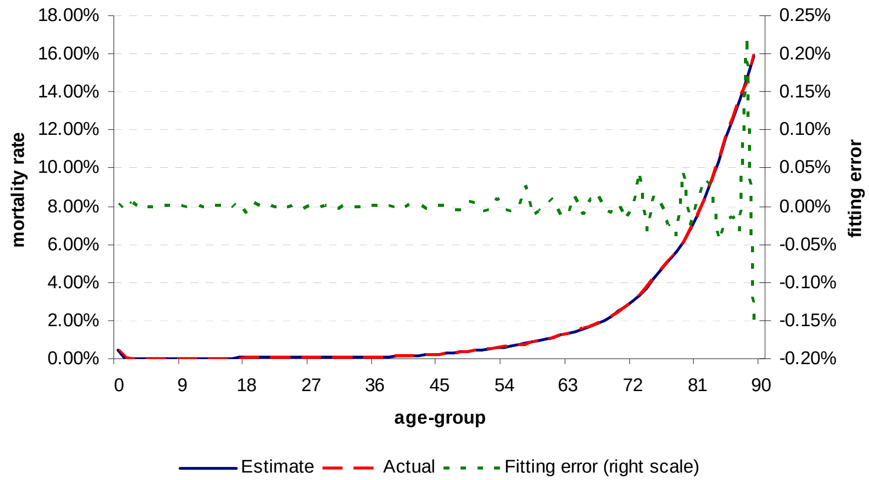
- Hazard rates are commonly used in survival analysis
- Weibull methodology is applied to fit the spot distribution
- Not of particular importance for suggested options pricing but might be of interest to find closed-form solution
- Weibull cdf: $\hat{F}(t) = 1 - e^{-\left(\frac{t}{T}\right)^b}$
  where $\hat{F}(t)$ = estimated mortality rate for age-group *t*
- Linearizing the cdf: $y = \ln\ln\frac{1}{1-\hat{F}(t)} = b\ln(t) - b\ln(T)$

  with 'characteristic lifetime' $T = \exp^{-\left(\frac{a}{b}\right)}$

  $\rightarrow$ Obtain estimates for *b* (slope) and *a* (intercept) through regression
- To get good fit, decompose spot distribution into small intervals (e.g. age-group [0;4], …, [85;89])
- Fitting error is defined as $\hat{F}(t) - F(t)$
  (*F(t)* = actual mortality rate)

Weibull fit, males (2005)

Percentage

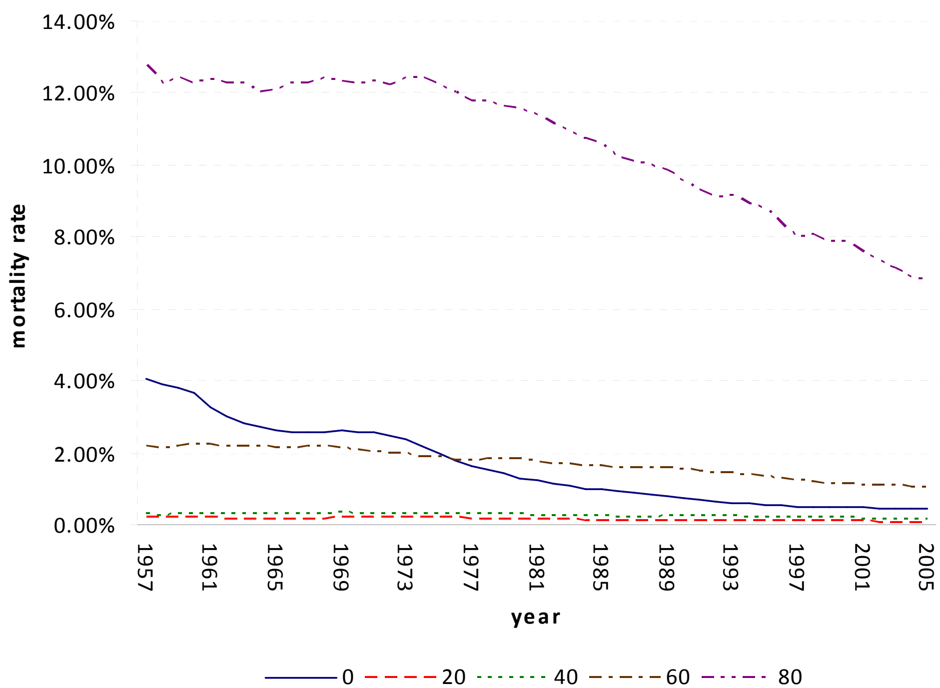


Source: Statistisches Bundesamt, own calculations

Description of the underlying

Stochastic trends in mortality rates

Trends in mortality rates for different age-groups (males, 1957 – 2005)
Percentage



Source: Statistisches Bundesamt

Main findings

- Decreasing mortality rates (i.e. mortality improvements)
- Different trends across age-groups:
  - Mortality rates of both infants and old aged males significantly decrease over time
  - Mortality rates of other age-groups are rather 'flat'
- Reasons:
  - "Base effect": mortality rate of say 10 year old males was already just 0.05% in 1957 (vs. 0.01% in 2005)
    → decreasing mortality rates are less significant in absolute terms
  - Factors such as advances in health care, improving living conditions, increasing wealth etc. affect age-groups differently
- **Mortality improvements are stochastic and vary substantially between age-groups**

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Description of the underlying

Summary

To model future mortality rates, development of the underlying can be decomposed into…

› ***Spot distribution*** (current mortality rates)

+

› ***Stochastic trend***

+

› ***Mean reverting jumps*** (to account for 'shocks' such as wars or pandemics)

+

› ***Slope dummies*** and / or ***non mean reverting jumps*** (to model instantaneous and significant mortality improvements)

=

› **Lévy process** (i.e. process that contains a drift, a diffusion and a jump component)

Options Pricing Framework

The forward issue

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The forward issue

Interest rate vs. longevity options

Interest rate options

- Arbitrage relationship between spot and forward market
- Forward interest rates are derived from current zero rates ("point estimates")
  → Arbitrage when forward diverges from point estimate
- One forward curve for each currency
- Standard market model for caps and floors: Black-76 model
  → IR options are priced on the basis of forward interest rates
- Accordingly, IR options are hedged via positions in the LIBOR forward market (e.g. futures contracts)
- Application of arbitrage-free pricing frameworks

Longevity options

- No arbitrage relationship between spot and forward market
- Forward mortality rates are derived through simulation
  → Simulated forwards as *best estimate* for future spot rates ("range of forwards")
  → Arbitrage when forward diverges from range
- Two forward surfaces per "region" (males & females)
- No standard model, but B76 potential benchmark
- As longevity options could be hedged by futures contracts (e.g. LifeMetrics q-Forwards framework), they can be priced on the basis of forward mortality rates
- *If* a liquid market for longevity futures evolves, *then* application of arbitrage-free pricing frameworks is feasible

Main difference: Determination of forward curves

Assumption for option pricing: Liquid futures market

Options Pricing Framework

The model

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The Model

Introduction

- Model is based on mortality rates, age-groups, gender
- Mortality rates follow a Lévy process
- Longevity options are priced on the basis of simulated forward mortality rates
- Assumption: Existance of a liquid futures market
- Risk-neutral valuation: Option value is obtained by discounting future cash flows at the risk-free rate of return
- *Put-call-parity* condition independent of price distribution; does it hold for the proposed options pricing framework?

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The model

Main option pricing families

	+	−
Closed-form solutions (e.g. Black-Scholes)	- Simple pricing formulas - Fast: Able to calculate large number of option prices and / or sensitivities in a very short time	- Based on strong assumptions (e.g. underlying asset returns normal distributed → realistic?) - Only calculates option price with fixed expiry date - Generally, does not allow for complex underlying processes and / or multiple input factors
Tree models (e.g. Binomial model)	- Recursive method → value options where holder has to make decisions prior to maturity, e.g. American options	- Relatively slow
Monte Carlo simulations	- Value options with multiple sources of uncertainty, complex underlying processes or complicated features - Very flexible	- Approximation technique - Time-consuming / slow - Only applied where analytical solutions do not exist

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The model

Monte Carlo simulation

Input parameters

- **Simulated forward mortality rate** $q_{(country,\ gender,\ age\text{-}group)}$
- **Volatility** $\sigma_{(country,\ gender,\ age\text{-}group)}$
- **Time to maturity** T
- **Risk-free interest rate** r
- **Strike** K

Pricing methodology

- **Mortality rates follow a stochastic process**
- **Process is stepwise replicated: Model 1 ('basic model'), 2 and 3 ('final model')**
- **Option value: Present value of average expected payoff, where**
  $payoff\ call_{(country,gender,age\text{-}group)} = max\ (q_{(T,x)} - K,\ 0)$ **and**
  $payoff\ put_{(country,gender,age\text{-}group)} = max\ (K - q_{(T,x)},\ 0)$
  with $q_{(T,x)}$ **= mortality rate of age-group** x **at maturity** T
- **To calculate the average expected payoff, a large number of random paths is simulated and their payoffs are averaged**

Pricing options is often referred to as the pricing of volatility.
Pricing longevity options means pricing volatility and the forward.

The model

Input parameter: Simulated forward mortality rates (1/2)

From a drift model to a Lévy process

- **Model 1:** Forward determination on the basis of the **spot distribution** as well as **age-group and gender specific trends**
  - Derive age-group specific slopes and intercepts for males and females from historical data through regression
- **Model 2:** *Model 1* is augmented to allow for **mean reverting jumps** to model ‘shocks’ (pandemics, wars etc.)
  - “One-sided“ jumps: Only increasing mortality rates
  - Mean reversion: Assume that mortality rates will be pulled back to predicted path as soon as ‘shock’ disappears
- **Model 3:** *Model 2* is augmented to model significant and instantaneous mortality improvements. Those effects can be captured via **non mean reverting jumps.** Alternatively: **Slope dummies** (or combination of both)
  - “One-sided“: Only decreasing mortality rates
  - No mean reversion: Assume that significant and instantaneous mortality improvements are perfectly memorized

Change in mortality rates dq: $dq = f * dt + \sigma * W + g_1 * J_1 + g_2 * J_2$

Where f = trend, dt = change in time, σ = volatility, W = random walk g_1 / g_2 = jump size jump 1 / jump 2, J_1 / J_2 = processes that count jumps

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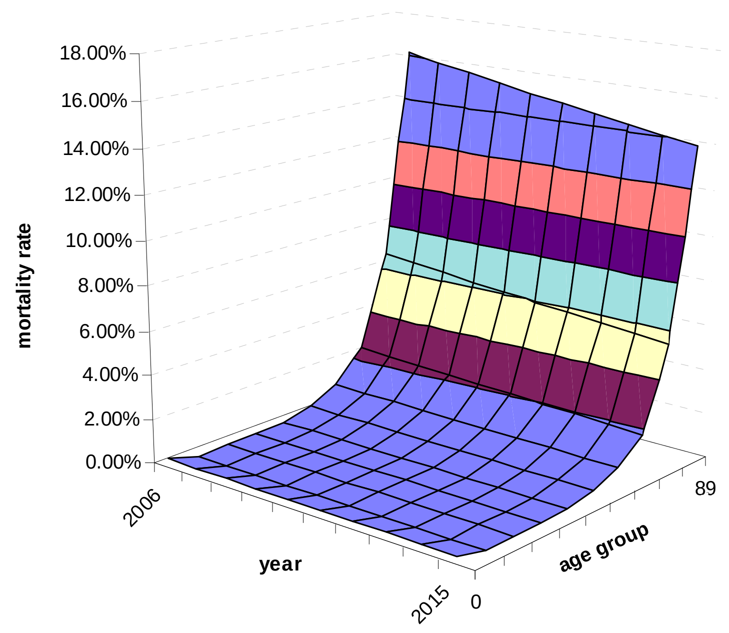
The model

Input parameter: Simulated forward mortality rates (2/2)

Example: Forward determination (model 1)

- Forwards for males and females are simulated separately
- Spot distribution: Most recently published mortality rates
- Exponential trend for each age-group is derived from say the previous 10 years through regression ($y = b * e^{a * x}$)
  → Age-group and gender specific intercept *b* and slope *a*
  → Note 1: Backtesting shows that exponential trend is superior to logarithmic or linear trend
  → Note 2: Backtesting confirms that mortality rates are difficult to forecast. Although prediction is relatively precise in many backtestings, extrapolation delivers poor results in case of significant trend changes
- Exponential trends not only most accurate but also reasonably logical (e.g. no negative mortality rates for long term forecasts)
- To forecast the say forthcoming 10 years, the spot distribution is extrapolated with the trend by assuming that mortality improvements will possess similar characteristics

Mortality rates surface, males (2006 - 2015)
Percentage

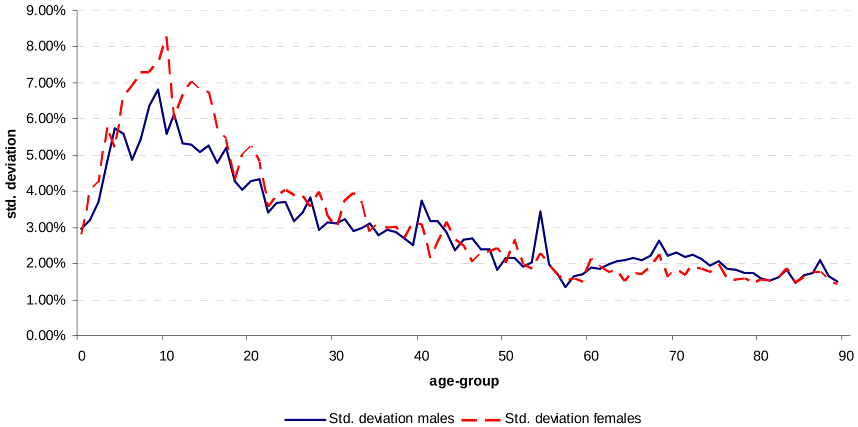


Source: Statistisches Bundesamt, own calculations

The model

Input parameter: Volatility

Age-group specific historical volatilities
Percentage



Source: Statistisches Bundesamt, own calculations

Volatility estimate

- Derived from historical data
- Gender and age-group specific historical volatilities are estimated as standard deviation of logarithmic changes in mortality rates from 1957 – 2005
- Volatility estimates appear to be quite low, e.g. compared to the volatility of interest rate options
- News usually causes volatility: Expect increasing volatility when market becomes liquid
- Options are priced with ‘flat’ volatility; usually volatility premiums are charged for “uncertainty” (e.g. longer maturities)
- Volatility surface potential area for further research

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The model

Input parameters

Summary	
Forward mortality rates *q*	Simulated
Volatility p.a. σ	Historical standard deviation (age-group and gender specific)
Time to maturity *T*	6 month 5 years 10 years
Risk-free rate of return *r*	5.00% p.a.
Strike *K*	Forward ATM 10% forward ITM / OTM
Age-groups	80, 60, 40, 20, 0 (males & females)
Models	1: Drift model 2: Model 1 is augmented to allow for **mean reverting jumps** to model 'shocks' 3: Model 2 is augmented to allow for **non mean reverting jumps** to model significant and instantaneous mortality improvements
Approach	MCS with 1,000 paths

The model

Model 1 (1/2)

Drift model

- Forward determination: Extrapolation of spot distribution with gender and age-group specific trend (obtained through regression)
- To simulate one random path, the Euler method is applied:

$$q_{(t+1,x)} = q_{(t,x)} \times \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t}\,\phi\right]$$

  where $q_{(t,x)}$ = (simulated) mortality rate at time t for age-group x, σ = volatility p.a., δt = time step (e.g. 1/365 for daily steps), ϕ = normally distributed random errors and μ = drift rate p.a.

- The drift rate is the sum of *r* and the age-group specific (negative) slope (i.e. disregarding *r* and stochastic, *q(t,x)* equals the forecasted mortality rate)
- The option value is calculated as present value of the average expected payoff at maturity (1,000 random paths)

Option pricing in excel

	A	B	C	D	DP	DQ	DR	DS	DT	DU	DV
1	**Simulation Input**										
2	age-group	80									
3	slope	-2.28%	p.a.								
4	r	5.00%	p.a.								
5	drift	2.72%	p.a.								
6	K	8.90%									
7	σ	1.60%	p.a.								
8	T	10	years								
9	Time steps	120	monthly								
10											
11		**Step 0**	**Step 1**	**Step 2**	**Step 118**	**Step 119**	**Step 120**	**Call Payoff**	**Put Payoff**		
12		**0.00**	**0.08**	**0.17**	**9.83**	**9.92**	**10.00**				
13	**Sim 1**	6.7833%	6.7935%	6.7699%	9.6235%	9.6712%	9.6869%	0.7867%	0.0000%		
14	**Sim 2**	6.7833%	6.8018%	6.8576%	8.3137%	8.3452%	8.4348%	0.0000%	0.4655%		
15	**Sim 3**	6.7833%	6.7530%	6.7578%	8.5019%	8.5198%	8.5972%	0.0000%	0.3030%		
1010	**Sim 998**	6.7833%	6.8037%	6.8593%	8.7752%	8.7056%	8.7219%	0.0000%	0.1783%		
1011	**Sim 999**	6.7833%	6.8042%	6.7987%	8.8310%	8.7600%	8.8654%	0.0000%	0.0349%		
1012	**Sim 1000**	6.7833%	6.8167%	6.8216%	8.8131%	8.8053%	8.9027%	0.0025%	0.0000%		
1013							**Average**	0.1779%	0.1755%		
1014							**PV**	0.1079%	0.1064%		
1015							**Stdev**	0.1622%	0.1583%		
1016							**Call 95% CI**	0.0978%	0.1179%		
1017							**Put 95% CI**	0.0966%	0.1162%		
1018											
1019							**Put price (PC parity)**		0.1079%	**PC HOLDS**	
1020											

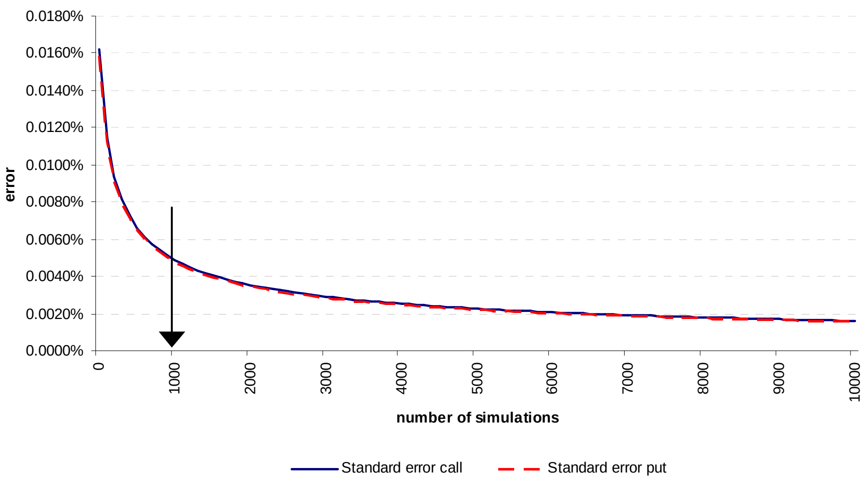
Source: Statistisches Bundesamt, own calculations

The model

Model 1 (2/2)

MCS standard error

Percentage



Source: Statistisches Bundesamt, own calculations

Evaluation

- MCS standard error
  → Increase number of simulations to improve precision
- Options gain in value as time to maturity increases
- Relative option premiums increase as a function of volatility (i.e. age-groups with a high volatility are relatively more expensive than those with low volatility)
- 10% ITM calls have approximately the same value as 10% ITM puts
- Put-call parity holds for chosen confidence level (95%)
- Drift model
  → Results converge against B76 option prices
  → Model 1 is a good benchmark the results of model 2 and 3 can be compared against

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The model

Model 2 (1/2)

Jump model I

- Model 1 is augmented to allow for mean reverting jumps to account for ‘shocks’ (e.g. wars, pandemics)
- Those jumps are i) one-sided (only increasing mortality rates) and ii) it is assumed that mortality rates are pulled back to their predicted path as soon as ‘shock’ disappears (mean reversion)
- To simulate one random path, a method described by Clewlow & Strickland is applied:

$$q_{(t+1,x)} = q_{(t,x)} \times \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t}\phi + \alpha(\psi_{(t,x)} - \lambda \times Y - \ln q_{(t,x)}) \times \delta t + Y \times p\right]$$

  where α = mean reverting intensity, λ = jump intensity, Y = jump size and p = Poisson distributed random variables. $\psi_{(t,x)}$ is the path the underlying will mean revert to (i.e. simulated mortality rates)

- Adjustment: Make mean reversion conditional to a previous jump (i.e. *if* jump in any previous period, *than* mean reversion, otherwise not)
- Note: As the dataset does not contain ‘shocks’, assumptions regarding the parameters mentioned above have to be made

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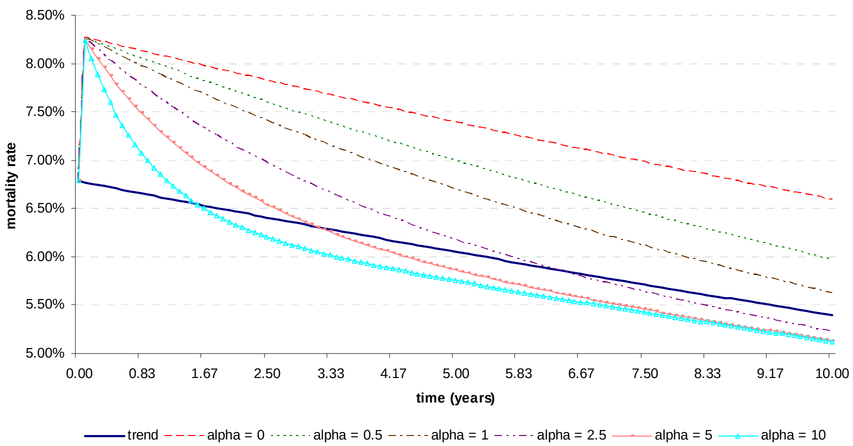
The model

Model 2 (2/2)

Mean reversion for different α (jump after 1 month)
Percentage



Source: Statistisches Bundesamt, own calculations

Evaluation

- For simplicity, jumps are assumed to be identical for all age-groups and no distinction is made between males and females, although this would not hold in practice
- The figure shows that the mean reverting process $\alpha(\psi_{(t,x)} - \lambda * Y - \ln q_{(t,x)}) * \delta t$ tends to overshoot, especially for high α
  → Unwanted property (it is illogical that jumps induce values below the trend thereafter)
  → Put values systematically too high
- One-sided jumps (only increasing mortality rates): Calls gain on the expense of decreasing puts
- Due to increasing stochastic (random Poisson variables) in conjunction with the asymmetric payoff of options, calls gain by more than puts lose in value
- This also causes an increasing standard error and thus decreasing pricing accuracy
- Put-call parity holds

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The model

Model 3 (1/2)

Jump model II

- Model 2 is augmented to allow for non mean reverting jumps to capture significant and instantaneous mortality improvements
- Those jumps are i) one-sided (only decreasing mortality rates) and ii) it is assumed that significant and instantaneous mortality improvements are perfectly memorized (no mean reversion)
- To simulate one random path, the method described by Clewlow & Strickland is extended as follows:

$$q_{(t+1,x)} = q_{(t,x)} \times \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)\delta t + \sigma\sqrt{\delta t}\phi + \alpha(\psi_{(t,x)} - \lambda \times Y - \ln q_{(t,x)}) \times \delta t + Y \times p - Y_2 \times p_2\right]$$

  where p_2 is determined on the basis of λ_2 (jump intensity of the second jump) and δt and Y_2 = jump size of the second jump; the negative sign accounts for decreasing mortality rates in case of a jump
- Note: Input parameters for second jump are not derived empirically; they are solely based on subjective assumptions

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The model

Model 3 (2/2)

Option prices (males, age-group 80 / 60 / 40 / 20)

Time to maturity	Strike	Call (MCS)	Put (MCS)	Put (PC parity)
Age-group 80, male, volatility 1.60% p.a. (flat) monthly time steps (10 years and 5 years) / daily time steps (6 month)				
10 years	8.9002%	0.1230%	0.1538%	0.1528%
	9.7902%	0.0242%	0.5700%	0.5713%
	8.0102%	0.5362%	0.0072%	0.0071%
5 years	7.7700%	0.1329%	0.1148%	0.1149%
	8.5470%	0.0224%	0.6094%	0.6094%
	6.9930%	0.6241%	0.0009%	0.0008%
6 month	6.8747%	0.0403%	0.0374%	0.0375%
	7.5622%	0.0028%	0.6748%	0.6747%
	6.1873%	0.6739%	0.0000%	0.0000%
Age-group 60, male, volatility 1.89% p.a. (flat) monthly time steps (10 years and 5 years) / daily time steps (6 month)				
10 years	1.3723%	0.0246%	0.0267%	0.0265%
	1.5095%	0.0046%	0.0911%	0.0910%
	1.2351%	0.0836%	0.0021%	0.0021%
5 years	1.1930%	0.0223%	0.0196%	0.0196%
	1.3123%	0.0037%	0.0938%	0.0940%
	1.0737%	0.0960%	0.0003%	0.0002%
6 month	1.0515%	0.0063%	0.0064%	0.0064%
	1.1567%	0.0001%	0.1036%	0.1035%
	0.9464%	0.1021%	0.0000%	0.0000%
Age-group 40, male, volatility 3.74% p.a. (flat) monthly time steps (10 years and 5 years) / daily time steps (6 month)				
10 years	0.1463%	0.0042%	0.0045%	0.0044%
	0.1609%	0.0014%	0.0106%	0.0107%
	0.1317%	0.0099%	0.0013%	0.0014%
5 years	0.1419%	0.0042%	0.0039%	0.0040%
	0.1560%	0.0010%	0.0117%	0.0115%
	0.1277%	0.0119%	0.0005%	0.0004%
6 month	0.1380%	0.0016%	0.0015%	0.0015%
	0.1518%	0.0000%	0.0136%	0.0137%
	0.1242%	0.0134%	0.0000%	0.0000%
Age-group 20, male, volatility 4.26% p.a. (flat) monthly time steps (10 years and 5 years) / daily time steps (6 month)				
10 years	0.0537%	0.0017%	0.0018%	0.0017%
	0.0590%	0.0007%	0.0040%	0.0041%
	0.0483%	0.0038%	0.0006%	0.0005%
5 years	0.0569%	0.0019%	0.0017%	0.0017%
	0.0626%	0.0005%	0.0048%	0.0049%
	0.0512%	0.0049%	0.0003%	0.0003%
6 month	0.0599%	0.0008%	0.0007%	0.0007%
	0.0659%	0.0000%	0.0059%	0.0059%
	0.0539%	0.0134%	0.0000%	0.0000%

Source: Statistisches Bundesamt, own calculations

Evaluation

› For simplicity, jumps are assumed to be identical for all age-groups and no distinction is made between males and females, although this might not hold in practice

› Compared to model 1, the forward is determined as average of final mortality rates $q_{(T,x)}$ across 1,000 simulations at r = 0.00% and σ = 0.00% (i.e. random Poisson variables are the only remaining stochastic)

› Comparing the results to those of model 1 and 2, they appear to be sensible:
  - Calls lose in value relative to puts
  - When setting $\lambda_1 = \lambda_2$, $Y_1 = Y_2$ and $\alpha = 0$, ATM puts ≈ ATM calls but the values are higher than those in model 1
    → Incorporation of random Poisson variables increases the standard deviation of option values
    → Increasing premiums

› Put-call parity holds

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Options Pricing Framework
Sensitivity analysis

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Sensitivity analysis

Methodology

› Estimates for MCS sensitivities are obtained by applying the technique of numerical differentiation $\left(\frac{\hat{f}^* - \hat{f}}{\Delta x}\right)$

where $\hat{f}^*$ = new value for the derivative, $\hat{f}$ = base case estimate and Δx = small change in the underlying variable

› Base case: *males*, *age-group 80* with following input: T = 5 years, $q_{(t=0,80)}$ = 6.78%, *slope* = −2.28% p.a., r = 5.00% p.a. and thus *drift* = 2.72% p.a. , K = 7.75%, σ = 15.00% p.a. ; mean reverting jump: λ_1 = 0.10, Y_1 = 20.00% and α = 0.50; non mean reverting jump: λ_2 = 0.20 and Y_1 = 5.00% (5,000 simulations)

› The ‘greeks’ are important for risk managers to evaluate the sensitivities of their books:
- *Delta*: Change in the value of an option with respect to changes in the **underlying**
- *Gamma*: Change […] with respect to changes in **delta** (i.e. convexity)
- *Vega:* Change […] with respect to changes in **volatility**
- *Theta*: Change […] with respect to changes in **time**
- *Rho*: Change […] with respect to changes in **interest rates**

› Analysis of sensitivity to other input parameters allows prioritizing areas of further research

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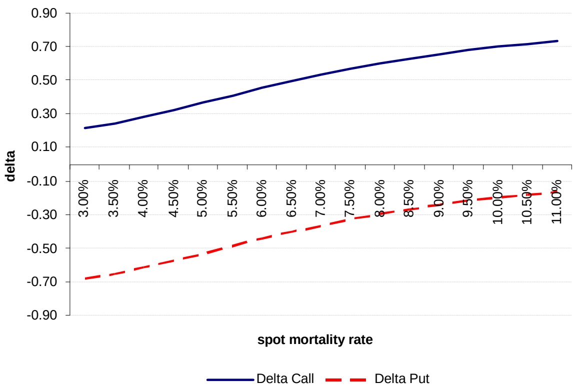
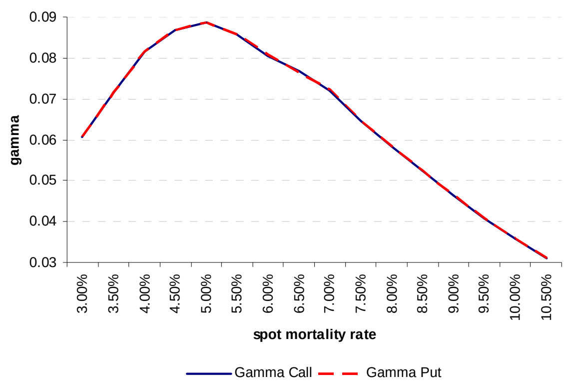
Sensitivity analysis

The greeks (1/2)

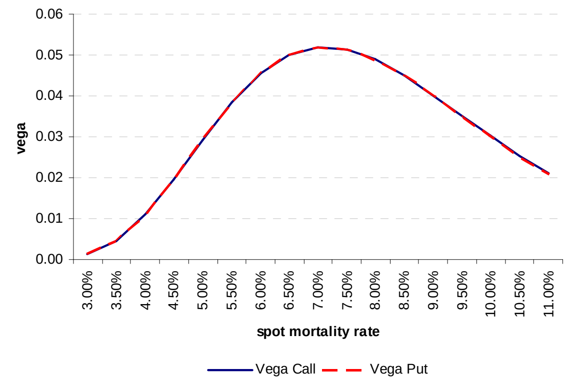
Delta
Euro



Gamma
Euro per Euro



Vega
per %



Source: Statistisches Bundesamt, own calculations

Delta

- Delta is ≈ 0.50 for ATM options
- Converges against 1 (-1) for ITM calls (puts) and against 0 for OTM options

Gamma

- MCS Gamma is least accurate as the error is amplified (1^{st} derivative of already erroneous Delta)
- For example, the peak is too far on the left as Gamma should be highest for ATM options (critical point → exercise / not exercise?)

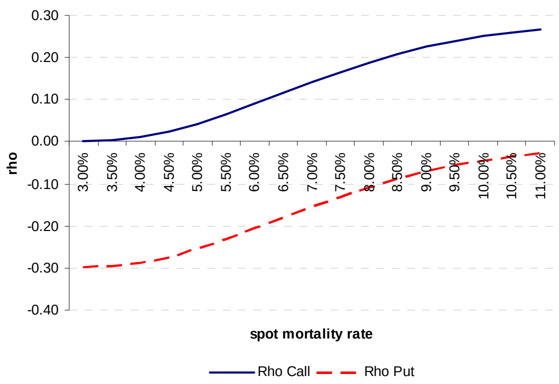
Vega

- Vegas are positive for puts and calls as both gain in value when volatility increases due to the asymmetric payoff of options
- Vega peaks ATM

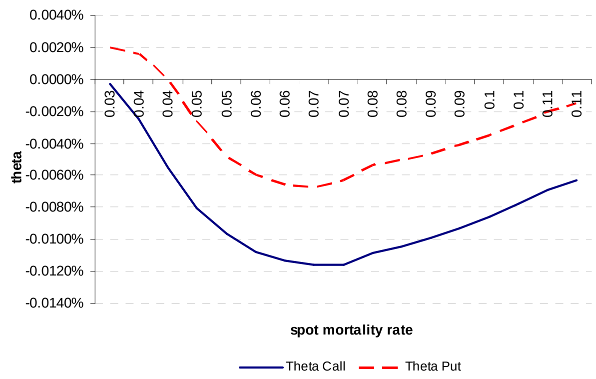
Sensitivity analysis

The greeks (2/2)

Rho
per %



Theta
per month



Source: Statistisches Bundesamt, own calculations

Rho

- Rho is positive (negative) for calls (puts) as they gain (lose) in value when interest rates increase
- It is larger for ITM options (due to cost of carry; ITM options require more cash) and decreases steadily as the option moves OTM

Theta

- Theta approaches 0 for OTM options (i.e. insensitive to changes in time)
- Time-decay is highest for ATM options
- Theta can be positive for ITM puts due to limited upside (mortality rates have a natural boundary of 0.00%)

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Sensitivity analysis

Other input parameters

Parameter	Call sensitivity	Put sensitivity
@ λ = 0.1050 **@ λ = 0.0950**	0.0074 0.0016	-0.0080 -0.0002
@ Y = 21.00% **@ Y = 19.00%**	0.0034 0.0034	-0.0015 -0.0015
@ α = 0.5250 **@ α = 0.4750**	-0.0008 -0.0008	0.0001 0.0001
@ ψ = -2.6994 **@ ψ = -2.7995**	0.0009 0.0008	-0.0004 -0.0005
@ λ2 = 0.2100 **@ λ2 = 0.1900**	-0.0010 -0.0002	0.0014 0.0026
@ Y2 = 5.25% **@ Y2 = 4.75%**	-0.0056 -0.0057	0.0059 0.0058

Evaluation

- Calls (puts) gain (lose) in value when either λ_1 or Y_1 increase → This jump (increasing mortality rates) by itself is one-sided
- Analogously, puts become more expensive on the expense of decreasing call prices when λ_2 or Y_2 increase
- When α is shifted upwards, mean reverting forces become stronger and hence call (put) premiums go down (up)
- An increasing ψ means that, after a shock, the underlying will mean revert to a higher level and thus promotes the value of calls whereas puts decrease in value
- Note: Lambdas are exposed to changing randomness in Poisson variables, diluting the findings
- **Conclusion: accurate estimates for λ_1, Y_1, λ_2 and Y_2 are more important than those for α and ψ**

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Conclusion

Conclusion

+	- High flexibility: → Easy to calibrate for country- and case-specific data as well as for individual expectations regarding future mortality rates → Easy transferable to other payoffs (e.g. Asian options) - Use of mortality rates is very intuitive from a capital markets perspective due to similarities with the credit world → Transfer applications from the credit to the longevity market (keyword *Structured products*)
−	- Overshooting – especially for high α - Subjective input parameters for jumps
Proposals for further research	- Volatility surface - Input parameter estimates: Empirically derive probability, amplitude and duration of jumps; distinguish also between age-groups and gender

Good starting point for further developments in longevity options

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Thank you for your attention!

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